Practice Problems Involving Induction.

1. Show that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

2. Show that if $x \geq -1$, then $(1 + x)^n \geq 1 + nx$.

3. Show that if $F_0 = 0$, $F_1 = 1$ and $F_{n+1} = F_n + F_{n-1}$, then $\left(\frac{3}{2}\right)^n \leq F_{n+2} \leq 2^n$.

4. Show that $\left(1 - \frac{1}{\sqrt{2}}\right)\left(1 - \frac{1}{\sqrt{3}}\right) \cdots \left(1 - \frac{1}{\sqrt{n}}\right) < \frac{2}{n^2}$.

5. Show that $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$.

6. Show that $\frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{n^2} < 2$.

7. Suppose that n lines are drawn in the plane so that no 2 lines are parallel and no 3 lines pass through the same point. Show that the lines divide the plane into $\frac{n^2 + n + 2}{2}$ regions.

8. There are n points in space where some pairs of points are joined by line segments. Suppose that the whole configuration of points and segments is connected, but if one segment is deleted then it will no longer be connected. Show that there must be $n - 1$ line segments.

9. Suppose that when you climb a flight of stairs you are able to take 1 step at time or 2 steps at a time. In how many different can you climb a staircase of n steps?