

HOMOLOGICAL ALGEBRA

HOMEWORK ASSIGNMENT I

Read pages 1-5, 417-424.

PROBLEMS

1. (Chriestenson, Gern, Harper) Exercise 1.1.6 of Weibel.
2. (Hower, Jones) Exercise A.1.3 of Weibel. (Change the word “isomorphic” to “equivalent”.)
3. (Keller, Li) First part of exercise A.1.4 of Weibel. (Be sure to check naturality.)
4. (Lizzi, Martinez) Let \mathcal{C} be the full subcategory of \mathbf{Ab} consisting of divisible groups. Show that the natural map $\mathbb{Q} \rightarrow \mathbb{Q}/\mathbb{Z}$ is a monic that is not 1-1.
5. (Strider, Moore, Moorhead) Let G and H be groups considered as 1-object categories.
 - (a) Show that functors between G and H correspond to group homomorphisms.
 - (b) For which pairs of functors $E, F: G \rightarrow H$ do there exist natural transformations $\eta: E \rightarrow F$?
6. (Pratarelli, Sanders) Let \mathcal{C} be the subcategory of \mathbf{Sets} containing all objects but only the identity morphisms. Let $F = G$ be the inclusion functor of \mathcal{C} into \mathbf{Sets} . Show that $\mathbf{Nat}(F, G)$ is not a set.
7. (Scherer, Selker, Tuley) Let O be the ordered pair functor from \mathbf{Sets} to itself ($O(X) = X^2$, $O(f) = f \times f$). Let U be the unordered pair functor from \mathbf{Sets} to itself. ($U(X) = \{\{x, y\} \mid x, y \in X\}$, $U(f)(\{x, y\}) = \{f(x), f(y)\}$. We allow $x = y$.) Find all natural transformations from O to U .
8. (Wakefield, Wiscons) Show that every small category can be embedded into the category of sets. (Hint: Use a functor that maps an object A to the set of morphisms with codomain A .)