Theorem 1. $\mathrm{Ext}^n(A,B) \cong \mathrm{ext}^n(A,B)$.

Proof. The proof is exactly like that for Tor, so we outline the steps only.

I. Start with projective and injective resolutions of A and B.

$$\begin{array}{ccccccccc} \cdots & \longrightarrow & P_1 & \longrightarrow & P_0 & \longrightarrow & A & \longrightarrow & 0 \\ 0 & \longrightarrow & B & \longrightarrow & E^0 & \longrightarrow & E^1 & \longrightarrow & \cdots \end{array}$$

II. Factor resolutions into SES's:

$$\begin{array}{ccccccccc} 0 & \longrightarrow & K_i & \longrightarrow & P_i & \longrightarrow & K_{i-1} & \longrightarrow & 0; \qquad A = K_{-1}. \\ 0 & \longrightarrow & L^{i-1} & \longrightarrow & E^i & \longrightarrow & L^i & \longrightarrow & 0; \qquad B = L^{-1}. \end{array}$$

III. Apply Hom (= H) to SES's to obtain a commutative diagram with exact rows and columns:

$$\begin{tikzcd} & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\ 0 \arrow[r] & H(K_{i-1}, L^{j-1}) \arrow[r] \arrow[d] & H(K_{i-1}, E^j) \arrow[r] \arrow[d] & H(K_{i-1}, L^j) \arrow[r] \arrow[d] & Z_{ij} \arrow[d] \\ 0 \arrow[r] & H(P_i, L^{j-1}) \arrow[r] \arrow[d] & H(P_i, E^j) \arrow[r] \arrow[d] & H(P_i, L^j) \arrow[r] \arrow[d] & 0 \arrow[d] \\ 0 \arrow[r] & H(K_i, L^{j-1}) \arrow[r] \arrow[d] & H(K_i, E^j) \arrow[r] \arrow[d] & H(K_i, L^j) \arrow[r] \arrow[d] & Y_{ij} \\ & W_{ij} \arrow[r] & 0 \arrow[r] & X_{ij} & \end{tikzcd}$$

IV. Use the Snake Lemma to deduce that $W_{ij} \cong Z_{ij}$. Use the defining property of cokernels and the commutativity of the small squares with lower right corners X_{ij} and Y_{ij} to deduce that $X_{ij} \cong Y_{ij}$.

V. Use the long exact sequences for Ext and ext and the facts that $\mathrm{Ext}^1(M, _)$ vanishes on injectives and $\mathrm{ext}^1(_, N)$ vanishes on projectives to deduce that

(i) $W_{ij} \cong \mathrm{ext}^1(K_{i-1}, L^{j-1})$
(ii) $X_{ij} \cong \mathrm{ext}^1(K_{i-1}, L^j)$
(iii) $Y_{ij} \cong \mathrm{Ext}^1(K_i, L^{j-1})$
(iv) $Z_{ij} \cong \mathrm{Ext}^1(K_{i-1}, L^{j-1})$

VI. Complete proof that $\mathrm{Ext}^n(A,B) \cong \mathrm{ext}^n(A,B)$ using dimension shifting and the isomorphisms $W \cong Z$ and $X \cong Y$. (Note: at this point of the proof we can only shift dimension in the right variable of Ext and the left variable of ext.)

Proof for $n = 1$: $\mathrm{Ext}^1(A,B) = \mathrm{Ext}^1(K_{-1}, L^{-1}) \cong Z_{00} \cong W_{00} \cong \mathrm{ext}^1(K_{-1}, L^{-1}) \cong \mathrm{ext}^1(A,B)$.

Proof for higher n:

$$
\begin{aligned}
\mathrm{Ext}^{n+1}(A,B) &= \mathrm{Ext}^{n+1}(K_{-1}, L^{-1}) \\
&\cong \mathrm{Ext}^1(K_{-1}, L^{n-1}) && \text{Dimension shifting for Ext} \\
&\cong \mathrm{ext}^1(K_{-1}, L^{n-1}) && Z_{0n} \cong W_{0n} \\
&\cong \mathrm{Ext}^1(K_0, L^{n-2}) && X_{0(n-1)} \cong Y_{0(n-1)} \\
&\cong \mathrm{ext}^1(K_0, L^{n-2}) && Z_{1(n-1)} \cong W_{1(n-1)} \\
&\cong \mathrm{Ext}^1(K_1, L^{n-3}) && X_{1(n-2)} \cong Y_{1(n-2)} \\
&\quad\vdots \\
&\cong \mathrm{ext}^1(K_{n-1}, L^{-1}) \\
&\cong \mathrm{ext}^{n+1}(K_{-1}, L^{-1}) && \text{Dimension shifting for ext} \\
&= \mathrm{ext}^{n+1}(A,B).
\end{aligned}
$$

$\square$