

# Assignment VI

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- 1 Calculate  $H^2(\mathbb{Z}_4, \mathbb{Z}_2)$ .

**Solution.** We apply Theorem 6.2.2, which tells us that

$$H^2(\mathbb{Z}_4, \mathbb{Z}_2) = \mathbb{Z}_2^{\mathbb{Z}_4} / N\mathbb{Z}_2,$$

where  $\mathbb{Z}_2^{\mathbb{Z}_4}$  is the subgroup of  $\mathbb{Z}_2$  fixed under the action of  $\mathbb{Z}_4$  (in other words,  $\mathbb{Z}_2$ , since the only automorphism of  $\mathbb{Z}_2$  is identity), and  $N = \varphi_0 + \varphi_1 + \varphi_2 + \varphi_3$  (as described in Calculation 6.2.1), where  $\varphi_j : \mathbb{Z}_2 \rightarrow \mathbb{Z}_2$  is the action of  $j \in \mathbb{Z}_4$ ; in this case, each is identity. Thus  $N0 = 0 + 0 + 0 + 0 = 0$  and  $N1 = 1 + 1 + 1 + 1 = 0$  as well, so  $N\mathbb{Z}_2 = (0)$ . Thus  $H^2(\mathbb{Z}_4, \mathbb{Z}_2) = \mathbb{Z}_2$ . ■