Assignment VI

Yingwei Li, John Tuley

1 *Calculate* $H^2(\mathbb{Z}_4, \mathbb{Z}_2)$.

Solution. We apply Theorem 6.2.2, which tells us that

$$H^2(\mathbb{Z}_4, \mathbb{Z}_2) = {\mathbb{Z}_2}^{\mathbb{Z}_4}/N\mathbb{Z}_2,$$

where ${\mathbb{Z}_2}^{\mathbb{Z}_4}$ is the subgroup of $\mathbb{Z}_2$ fixed under the action of $\mathbb{Z}_4$ (in other words, $\mathbb{Z}_2$, since the only automorphism of $\mathbb{Z}_2$ is identity), and $N = \varphi_0 + \varphi_1 + \varphi_2 + \varphi_3$ (as described in Calculation 6.2.1), where $\varphi_j : \mathbb{Z}_2 \to \mathbb{Z}_2$ is the action of $j \in \mathbb{Z}_4$; in this case, each is identity. Thus $N0 = 0 + 0 + 0 + 0 = 0$ and $N1 = 1 + 1 + 1 + 1 = 0$ as well, so $N\mathbb{Z}_2 = (0)$. Thus $H^2(\mathbb{Z}_4, \mathbb{Z}_2) = \mathbb{Z}_2$. ∎