# HOMOLOGICAL ALGEBRA: HOMEWORK 4

MATTHEW MOORE AND MICHAEL MARTINEZ

**1)** Suppose $T$ is a right exact functor and some derived functor $L_nT$, $n > 0$, is also right exact. Show that $L_mT$ is the zero functor for all $m \geq n$.

*Proof.* $T$ is right exact, so the left derived functor $L_nT$ is defined. The proof shall be by induction on $m$. Let $A$ be arbitrary and let $P_\bullet \to A$ be a projective resolution with factors $0 \to K_i \to P_i \to K_{i-1} \to 0$. Since $L_nT$ is right exact and $0 \to K_0 \to P_0 \to A \to 0$ is short exact, $L_nT(K_0) \to L_nT(P_0) \to L_nT(A) \to 0$ is exact. $P_0$ is projective and $n > 0$, so $L_nT(P_0) = 0$. Thus $L_nT(A) = 0$, proving the base case.

Suppose that $L_mT(A) = 0$ for all $A$. Then by dimension shifting, $L_{m+1}(A) = L_mT(K_0) = 0$. □

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