

## SET THEORY (MATH 4730)

### SUMMARY OF TOPICS FROM 1/12/09-3/6/09

- I. Axiomatic Set Theory
  - (a) Russell's Paradox.
  - (b) Valid constructions of new sets (pairing, union, power set, comprehension, intersection).
  - (c) Empty set, successor of a set.
- II. Relations and Functions
  - (a) Ordered pairs, product sets.
  - (b) Relations.
  - (c) Functions, equivalence relations, partitions, kernel, image.
  - (d) Ordered sets, linear orders, well-orders.
- III. Natural numbers
  - (a) Definition of  $\mathbb{N}$ .
  - (b) Induction.
  - (c)  $\mathbb{N}$  is well-ordered.
  - (d) Recursion Theorem.
  - (e) Arithmetic of  $\mathbb{N}$ .
- IV. Cardinality of sets
  - (a) Equipotence ( $|A| = |B|$ ).  $|A| \leq |B|$ ,  $|A| \leq m$ , etc.
  - (b) Cantor-Bernstein Theorem.
  - (c) Finite sets and their properties.
  - (d) Countable sets.
  - (e) Uncountable sets.
  - (f) Cantor's Theorem.
  - (g) Cardinality of  $\mathbb{N} \times \mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ ,  $\mathbb{R}^n$ ,  $(0, 1)$ ,  $\mathcal{P}(\mathbb{N})$ .

### General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

- (i) Know the definitions of new concepts, and the meanings of the definitions.
- (ii) Know the statements and meanings of the major theorems.
- (iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
- (iv) Know how to perform the different kinds of calculations discussed in class.
- (v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
- (vi) Know how to correct mistakes made on old HW.

**Sample Problems.**

- (1) Explain why the intersection of two sets is a set.
- (2) Prove or disprove:
  - (a)  $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$ .
  - (b)  $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$ .
- (3) What is a function? (Give the definition.)
- (4) What is the kernel of the successor function?
- (5) Explain why induction is a valid form of proof. (Your explanation should make use of the fact that  $\mathbb{N}$  is a subset of every inductive set.)
- (6) Show that  $m + n = 0$  implies  $m = n = 0$  for natural numbers  $m$  and  $n$ . Then show that  $m + n = 1$  implies that  $m$  and  $n$  are 0 and 1 in some order.
- (7) Show that the set of irrational real numbers is uncountable.
- (8) Are there uncountably many ways to well-order  $\mathbb{N}$ ?