DISCRETE MATH (MATH 2001)

SUMMARY OF TOPICS FROM 1/12/09-3/6/09

I. Set Theory (Notes, Sections 1.3-1.4)
- (a) Informal notion of a set. The axioms.
- (b) Valid constructions of new sets (pairing, union, power set, separation, intersection)
- (c) Empty set, successor of a set.
- (d) Inductive sets, natural numbers.

II. Induction (Section 1.2)
- (a) Ordinary induction.
- (b) Strong induction.

III. Functions (Section 3.1)
- (a) Ordered pairs, $A \times B$, definition of a function, definition of a sequence.
- (b) Injections, surjections, bijections.

IV. Recursion (Section 1.5)
- (a) Recursively defined functions. Recursion Theorem (statement only).
- (b) Factorials, Fibonacci numbers.
- (c) Recursive definitions of arithmetic operations on $\mathbb{N}$: $x + y, xy, x^y$.
- (d) Use of induction to prove laws of arithmetic.

IV. Cardinality (Section 3.2)
- (a) Definitions of $|A| = |B|$, $|A| \leq |B|$, $|A| = m$, etc.
- (b) Cantor-Schröder-Bernstein Theorem (statement only):
  if $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.
- (c) Sum Rule, Product Rule.

V. Binomial coefficients (Section 1.1)
- (a) Definition.
- (b) Pascal's Triangle, Pascal's Identity.
- (c) Binomial Theorem.
- (d) Counting Arguments.

VI. Multinomial coefficients (Pages 102-103, plus class notes)
- (a) Definition.
- (b) Pascal's Pyramid, Pascal's Identities in higher dimensions.
- (c) Multinomial Theorem.
- (d) Ordered Partitions.

VII. Unordered partitions and equivalence relations (Section 3.1)
- (a) Definitions.
- (b) Bell numbers, recursion for Bell numbers.

   (c) Partition Theorem.
   (d) Use of equivalence relations to construct $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{Z}_n$.

VIII. Arithmetic of $\mathbb{Z}$. (Sections 2.1 and 2.3)
   (a) Every positive integer is a product of primes.
   (b) The nonnegative integers (or natural numbers) are well-ordered.
   (c) Defn of gcd, lcm. $\gcd(a, b) \cdot \mathrm{lcm}(a, b) = a \cdot b$.
   (d) Division algorithm, Euclidean algorithm.
   (e) The gcd of a and b is an integral combination of of a and b. (Be able to find such a combination.)
   (f) Euclid's Lemma.
   (g) Determine the solvability (and solutions) of linear diophantine equations in 1 or 2 variables ($ax = b$ or $ax + by = c$).

General advice on preparing for a math test.
Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.

Sample Problems.

(1) What is a function? (Give the definition.)
(2) Show that $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.
(3) Explain why induction is a valid form of proof. (Your explanation should make use of the fact that $\mathbb{N}$ is a subset of every inductive set.)
(4) Prove that $m(n + k) = (mn) + (mk)$ for all $m, n, k \in \mathbb{N}$.
(5) Show that the cardinality of the real interval $[0, 1]$ is the same as the cardinality of the interval $[1, 10]$.
(6) Which is larger, the nth Bell number or the nth Fibonacci number?
(7) Find $\gcd(1001, 168)$. Find $x, y \in \mathbb{Z}$ such that $1001x + 168y = \gcd(1001, 168)$.
(8) Show that $4x + 9y = n$ is solvable over $\mathbb{Z}$ for all n.