

# DISCRETE MATHEMATICS

## HOMEWORK ASSIGNMENT I

Read Sections 1.3 and 1.4.

### PROBLEMS

1. Consider the following two properties: (a)  $A \in B$ , and (b)  $A \subseteq B$ .
  - (i) Give examples of sets  $A$  and  $B$  where property (a) holds but (b) fails.
  - (ii) Give examples of sets  $A$  and  $B$  where property (a) fails but (b) holds.
  - (iii) Give examples of sets  $A$  and  $B$  where properties (a) and (b) both hold.
  - (iv) Give examples of sets  $A$  and  $B$  where properties (a) and (b) both fail.
  - (v) Explain why it is that if  $A$  and  $B$  are distinct natural numbers, then both (a) and (b) hold or both (a) and (b) fail.
2. A set  $T$  is *transitive* if  $R \in S \in T$  implies  $R \in T$ .
  - (i) Give an example of a set that is transitive.
  - (ii) Give an example of a set that is not transitive.
  - (iii) Show that  $A$  is transitive iff  $A \subseteq \mathcal{P}(A)$ .
3. (Ordered pairs) If  $A$  and  $B$  are sets, let  $(A, B) := \{\{A\}, \{A, B\}\}$ . The set  $(A, B)$  is called an *ordered pair*. Explain why
  - (i)  $(A, B)$  is a legitimate set. (That is, show that the axioms of set theory guarantee that if  $A$  and  $B$  are sets, then so is  $(A, B)$ .)
  - (ii)  $(A, B) = (C, D)$  iff  $A = C$  and  $B = D$ .