## The Axioms of Set Theory.

### Equality

(1) (Extentionality) Two sets are equal if they have the same elements.

### Existence of Special Sets

(2) (Empty Set) There is a set with no elements.
(Call it the emptyset and denote it by $\emptyset, \{\ \}$ or 0.)

(3) (Infinity) There is an inductive set.
(Equivalently, there is a set containing $\omega = \{0, 1, 2, \ldots\}$.)

### Creation of New Sets

(4) (Pairing) If $A$ and $B$ are sets, then $\{A, B\}$ is a set.

(5) (Union) If $I$ is a set, and $A_i$ is a set for each $i \in I$, then $\bigcup_{i \in I} A_i$ is a set.

(6) (Power Set) If $A$ is a set, then $\mathcal{P}(A)$ is a set.

(7) (Comprehension) If $A$ is a set and $P$ is a property given by a formula, then $\{x \in A \mid P(x)\}$ is a set.

(8) (Replacement) If $A$ is a set and $F$ is a function given by a formula, then $\{F(x) \mid x \in A\}$ is a set.

(9) (Choice) If $\{A_i \mid i \in I\}$ is set of nonempty disjoint sets, then there is a set $C$ such that $|A_i \cap C| = 1$ for every $i$.

### Sets have Special Properties

(10) (Foundation) There is no infinite sequence of sets $\cdots A_3 \in A_2 \in A_1 \in A_0$.