The Axioms of Set Theory.

The axioms of set theory can be found scattered throughout our book. Unfortunately they are not numbered, and we are not told how many there are, so it is difficult to determine the complete list of axioms by searching the book.

Here they are on one page. The ordering below is different from the ordering in the book for the following reason. The goal in the book is to introduce a few axioms early (Axioms (1), (2), (4)—(7)) to allow the development of the basic notions about relations and functions. But the goal of this handout is to group axioms together according to their function in order to obtain a good perspective on the subject of set theory.

Equality

(1) (Extentionality) Two sets are equal if they have the same elements.

Existence of Special Sets

(2) (Empty Set)[1] There is a set with no elements.
(Call it the *emptyset* and denote it by $\emptyset, \{\ \}$ or 0.)

(3) (Infinity) There is an inductive set.
(Equivalently, there is a set containing $\omega = \{0, 1, 2, \ldots\}$.)

Creation of New Sets

(4) (Pairing) If A and B are sets, then $\{A, B\}$ is a set.

(5) (Union) If I is a set, and A_i is a set for each $i \in I$, then $\bigcup_{i \in I} A_i$ is a set.

(6) (Power Set) If A is a set, then $\mathcal{P}(A)$ is a set.

(7) (Comprehension) If A is a set and P is a property given by a formula, then $\{x \in A \mid P(x)\}$ is a set.

(8) (Replacement) If A is a set and F is a function given by a formula, then $\{F(x) \mid x \in A\}$ is a set.

(9) (Choice) If $\{A_i \mid i \in I\}$ is set of nonempty disjoint sets, then there is a set C such that $|A_i \cap C| = 1$ for every i.

Sets have Special Properties

(10) (Foundation) There is no infinite sequence of sets $\cdots A_3 \in A_2 \in A_1 \in A_0$.

[1]Our book calls this the "Axiom of Existence" because, in the presence of the Axiom (7), Axiom (2) is equivalent to the statement "There exists a set".