Definitions and Laws of Arithmetic on $\mathbb{N}$.

Addition

$$\begin{aligned} m+0 &:= m && \text{(IC)} \\ m+S(n) &:= S(m+n) && \text{(RR)} \end{aligned}$$

Multiplication

$$\begin{aligned} m\cdot 0 &:= 0 && \text{(IC)} \\ m\cdot S(n) &:= m\cdot n+m && \text{(RR)} \end{aligned}$$

Exponentiation

$$\begin{aligned} m^0 &:= 1 && \text{(IC)} \\ m^{S(n)} &:= m^n\cdot m && \text{(RR)} \end{aligned}$$

Laws of successor. (These should be proved first.)

(a) 0 is not a successor.
(b) Successor is injective. ($S(m)=S(n)$ implies $m=n$.)

Laws of addition. (Provable by induction or from the definitions.)

(a) $S(n)=n+1$
(b) (Associative Law) $m+(n+k)=(m+n)+k$
(c) (Unit Law for 0) $m+0=0+m=m$
(d) (Commutative Law) $m+n=n+m$
(e) (Irreducibility of 0) $m+n=0$ implies $m=n=0$.
(f) (Cancellation) $m+k=n+k$ implies $m=n$.

Laws of multiplication (and addition).

(a) (Associative Law) $m\cdot(n\cdot k)=(m\cdot n)\cdot k$
(b) (Unit Law for 1) $m\cdot 1=1\cdot m=m$
(c) (Commutative Law) $m\cdot n=n\cdot m$
(d) (0 is absorbing) $m\cdot 0=0\cdot m=0$
(e) (Irreducibility of 1) $m\cdot n=1$ implies $m=n=1$
(f) (Distributive Law) $m\cdot(n+k)=(m\cdot n)+(m\cdot k)$

Laws of exponentiation (and multiplication and addition).

(a) $m^0=1$, $m^1=m$, $0^m=0$(if $m>0$), and $1^m=1$.
(b) $m^{n+k}=m^n\cdot m^k$
(c) $(m\cdot n)^k=m^k\cdot n^k$
(d) $(m^n)^k=m^{n\cdot k}$