Modern Algebra 2 (MATH 6140)
Test #2

This exam is due Friday, March 21. You are expected to complete three problems, one from each group. Clearly mark which problems are to be graded.

You may use your book, but you may not communicate with others concerning the exam. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

I have neither given nor received aid on this exam.

Name:____________________________

Group 1.

1. Let R be an integral domain. Suppose that every finitely generated R-module is isomorphic to one of the form $R/(a_1) \oplus \cdots \oplus R/(a_k) \oplus (\oplus^r R)$. Show that R is a PID.

2. Prove or disprove: the rational canonical form of a permutation matrix is a permutation matrix.

Group 2.

3. Find the rational canonical form and the Jordan canonical form of

$$\begin{bmatrix} 2 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

4. Show that the functor $V \mapsto V^*, \varphi \mapsto \varphi^*$ is an exact functor from the category of $\mathbb{F}$-spaces to itself.

Group 3.

5. Let R be the subring of $\mathbb{Q}$ whose elements are $\left\{\frac{m}{n} \in \mathbb{Q} \mid n \text{ odd}\right\}$. R is a PID. Describe the finitely generated torsion R-modules, and show that that are finite.

6. Let R be an integeral domain. For an R-module M and an element $r \in R$, let $M[r] = \{m \in M \mid rm = 0\}$. Show that the mapping $M \mapsto M[r]$ is the object part of a representable functor from the category of R-modules to itself.