Practice Sheet C

1. Call a positive integer N *persistent* if every positive multiple, kN, contains each of the digits $0, 1, \ldots, 9$ in its decimal expansion. Determine whether persistent integers exist, and if they do then exhibit the smallest one.

2. On a circle, n points are selected and the chords they determine are drawn. Assume that no three chords intersect (except at their endpoints). How many points of intersection of chords lie in the interior of the circle?

3. Does there exist a sequence of 1,000,000 consecutive positive integers, none of which is square-free?

4. Show that, if $2n$ points in space are joined by $n^2 + 1$ segments, then at least one triangle is formed. Show that it is possible to join $2n$ points in space with n^2 segments so that no triangle is formed.

5. Assume that the complex numbers $a_1, a_2, \ldots$ satisfy $|a_i - a_j| > 1$ for $i \neq j$. Show that $\displaystyle\sum_{i=1}^{\infty} \frac{1}{a_i^3}$ converges.

6. Which nonempty subsets $S \subseteq \mathbb{Z}^+$ have the property that all but finitely many sums of elements of S (possibly with repetitions) are composite integers?

7. Find a cubic polynomial whose zeroes are the cubes of the roots of $x^3 + ax^2 + bx + c = 0$