Review of “The shape of congruence lattices”
by
Keith Kearnes and Emil Kiss

The monographs by Freese and McKenzie (Commutator theory for congruence modular varieties) and by Hobby and McKenzie (The structure of finite algebras) have exerted a huge influence on the development of universal algebra and have been used to educate a generation (or two) of researchers in the field. The influence of these two books on the authors of the book under review is clear, and as the authors note, solving one of the problems posed in Hobby-McKenzie prompted them to write their book. This book contains much much more than the solution of this problem (the congruence lattice identity problem). What it primarily does is introduce and develop a new set of tools and ideas that are bound to significantly influence the field for years to come. Along with Freese-McKenzie and Hobby-McKenzie, it will be required reading for any serious student or practitioner of universal algebra. What Kearnes and Kiss have accomplished in this book is very impressive.

For many researchers in the field, tame congruence theory has become a standard tool, but some (including this reviewer) have become, perhaps, a little too reliant on it. The technology developed in this book provides a new perspective and a new set of tools for solving problems and will become part of the standard universal algebra toolkit. The final chapters of this book convincingly demonstrate the power of these new tools for solving problems and extending old results and it seems inevitable that further breakthroughs will be made with them.

The book is very well written and organized and I see no need for any major revisions. It is clear that the authors put a lot of thought and effort in to the writing of this book and I very much approve of the stylistic choices that they have made. Except for a couple of isolated cases (that have been noted), the proofs are correct and well structured. I did find a fair number of minor errors that I've noted in the attached pdf file. Rather than compile a long list of brief comments, I've elected to markup the pdf file with my comments (about 100 of them); these comments should be readable using any standard pdf reader and it should be possible, using the latest version of the free adobe reader, to alter or add to the comments. I've also attached a list of these comments in a separate pdf file (probably not too useful). A few of the errors that I found could have been picked up by running the file through a spell checker and/or by another careful proof-reading of the manuscript.

I suggest that the authors have someone else re-read the manuscript, not necessarily for mathematical correctness, since there are bound to be other typing errors that have not been uncovered.

I do have two suggestions that will take some effort to implement. The first is to introduce, at the end of the book, a list of open problems. A number of open problems are scattered throughout the book and it would be very helpful to gather these together in a list at the end, and to augment this list with other problems that may admit a solution using the ideas and techniques introduced in the book (for example, Pixley's ACD problem could be included and I am sure that other good ones could be generated). Of course I have in mind the list of problems that occur at the end of Hobby-McKenzie and the effect that this list has had on the development of the theory. I think that doing something similar in the book under review will provide an important service to the community.

The authors manage to avoid referring directly to the tame congruence theoretic types until page 191 (unless I've missed an earlier reference). [Not quite like writing a novel without using the vowel "e".] I suggest adding another section to Chapter 2 that provides a brief overview of tame congruence theory and at least gives some idea of what is meant by type i, for i one of the 5 possible types, as well as what it means for a minimal set to have an empty tail. A number of the main (and minor) theorems in the book are generalizations of results found in Hobby-McKenzie or proved using tame congruence theory, and it would be useful to provide, in some instances, brief descriptions of the earlier results. For example, before (or after) the statements of theorem 3.12 and/or 6.17 you could add that in the locally finite case, the conditions of the theorem are equivalent to the variety omitting type 1. Adding comments similar to this for some of the other generalizations of the "type omitting" theorems will help to place the new results in a context that many of us are familiar with and so will lead to a better appreciation of them.

In addition to the comments added to the pdf file, I've a few other, lengthier ones as well:

Chapter 1: Introduction

Section 1.3: Can a general definition or description be given of what is meant by a commutator theory? Near the end of the section, it is claimed that there are many other commutator theories but that they are approximations to the four described in detail. It would be helpful to explain in what sense these others are approximations, and also why all other commutator theories have this character.

Chapter 2: Preliminary notions

Section 2.3: Being an "old school" universal algebraist, I'm inclined to skip over anything that deals with adjoints, forgetful functors and other category theoretic terminology. The later material on meet continuous identities certainly justifies including this section in chapter 2 and so I'm not suggesting that anything drastic needs to be done here (the section is less than 2 pages, after all), but if you have any sympathy for an unrepentant category theory-phobe, you could consider providing a little more detail on what natural bijections, adjoints, and functors are.

Section 2.5: As noted on page 33, some of the details of the proof of Theorem 2.19 could be left to the reader. The same could be said for some of the other proofs in the manuscript.

Chapter 4: Meet continuous congruence identities

Section 4.4: I think that lemma 4.21 is not correct. It is probably not worth trying to fully repair it, since you only need to show that there are arbitrarily large finite simple nonmodular lattices of height 3.

Chapter 6: A theory of solvability

Section 6.3: As noted in the manuscript, it would be helpful to introduce a lemma before theorem 6.36 that establishes that above every congruence below 1_A there is a prime congruence.

Front/End matter:

I didn't really carefully read over the table of contents, the bibliography, and the index (but somebody should). I do indicate a couple of changes that could be made to them.