Let Φ be a commutative ring containing $\mathbb{Z}$. Let $a_1,\ldots,a_n$ be a basis of an algebra A (not necessarily associative) over Φ. (A can be the integers, the reals, the complex numbers, the quaternions, Cailey numbers, matrices,... .) Addition, multiplication and subtraction are performed on A^A pointwise. Theorem. Given two words $u(x_1,\ldots,x_k)$, $v(x_1,\ldots,x_k)$, built up from $x_1,\ldots,x_k$ by addition, subtraction, multiplication and composition, one can construct specific functions f_{ij}, $1 \leq i \leq k$, $1 \leq j \leq n$, on the integers such that $u = v$ is an identity on A^A iff the values of $u(f_1,\ldots,f_k)$ and $v(f_1,\ldots,f_k)$ at $a_1 + \ldots + a_n$ coincide. Corollary. The set of all identities on A^A is primitive recursive. $f_i(\sum_{j=1}^n z_j a_j) = \sum_{j=1}^n f_{ij}(z_j) a_j$, $1 \leq i \leq k$. (Received October 24, 1972.)

701-08-4. DAVID KELLY, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Basic equations: word problems and Mal'cev conditions.

An equation $p = q$ is basic if at most one nonnullary operational symbol appears in each of p and q. S and T always denote sets of basic equations. Theorem 1. The word problem for any finite set of basic equations is solvable. An equational class K admits the set D_n (resp., M_n, P_n) of basic equations iff K is n-distributive (resp., n-modular, n-permutable) [see R. Wille, "Kongruenzklassengeometrien", Springer-Verlag, 1970]. Since D_n, M_n and P_n are K-sets of length n, a question of G. Grätzer [J. Combinatorial Theory 8(1970), 334-342] is answered by Theorem 2. If S and T are K-sets with S of greater length than T, then there is an equational class that admits S without admitting T. Let S be in variables X with nullary operational symbols C. The proofs use our description of the free algebra of S in terms of the weak closure of S which is the smallest set T of basic equations containing S such that: (1) $p = q \in T$ implies $q = p \in T$; (2) $p = q$, $q = r \in T$ implies $p = r \in T$; (3) $p = q \in T$ and $\varphi: X \to X \cup C$ implies $p\varphi = q\varphi \in T$; (4) $p = q$, $f = g \in T$ for $f, g \in C$ implies $p = p' \in T$ where one occurrence of f in p is replaced by g to form p'. (Received November 13, 1972.)

701-08-5. ROBERT M. VANCKO, Ohio University, Athens, Ohio 45701. P-independence.

G. Grätzer defined weak independence as a general notion of independence which when specialized to abelian groups would yield the classical linear independence. Here, a more general notion of independence is defined which also specializes to linear independence when applied to abelian groups. Let $\mathfrak{A} = \langle A; F\rangle$, $\mathfrak{B} = \langle B; F\rangle$ be algebras, $T \subseteq A$. A mapping $\varphi: T \to B$ is primary if it can be extended to a homomorphism of $[T]_{\mathscr{P}^{(1)}(\mathfrak{A})}$ into B, where $\mathscr{P}^{(1)}(\mathfrak{A})$ is the set of unary polynomials of $\mathfrak{A}$ and $[T]_{\mathscr{P}^{(1)}(\mathfrak{A})}$ is the closure of T in the reduct $\langle A; \mathscr{P}^{(1)}(\mathfrak{A})\rangle$. $I \subseteq A$ is P-independent if every primary map $\varphi: I \to A$ can be extended to a homomorphism of $[I]_{\mathfrak{A}}$ into A. There are P-independent sets which are not weak independent. Theorem 1. A subset I of an abelian group is P-independent iff it is linearly independent. Analogous to a result previously proved by the author for weak independence: Theorem 2. The class of algebras in which the property ($\{a_i \mid i \in I\}$ is P-independent iff $[\{a_i \mid i \in I\}] = \sum_{i \in I}[a_i]$) holds is precisely the class of semimodules over semirings. (Received November 13, 1972.)

701-08-6. STEPHEN D. COMER, Vanderbilt University, Nashville, Tennessee 37235. Sectional representations induced by L-restricted direct decompositions. Preliminary report.

Assume that L is an ideal in P(I) containing all finite subsets of I. An isomorphism of an algebra A with an L-restricted product of nontrivial algebras induces in a natural way a set B of factor relations which form a BA under relative product and intersection. The sectional representation obtained from B using the standard

for every maximal ideal M in C, $\lambda(M) = A^{\angle}$ if $\mu^{-1}(M) \equiv L$; (2) for every $\psi \in \mu(L)$ and $f: I(-\psi) \to A$, $\bigcap\{f(i)/\lambda(i): i \in I(-\psi)\} \neq 0$. As corollaries we obtain characterizations of representations induced by weak and strong direct product decompositions. (Received November 13, 1972.)

10 Number Theory

*701-10-1. LAL M. CHAWLA, Kansas State University, Manhattan, Kansas 66506. A note on distributive arithmetical functions. Preliminary report.

We define an arithmetical function F(n) to be distributive if $(m, n) = 1$ implies $F(mn) = \theta(m)F(n) + F(m)\theta(n)$, where $\theta(n)$ is a multiplicative arithmetical function such that $\theta(n) \neq 0$, $n \geq 1$. F(n) is called completely distributive if the above equality holds for all positive integers m, n where $\theta(n)$ is a completely multiplicative function. In this note we prove that an arithmetical function F(n) is (completely) distributive if and only if $F(n) = \theta(n)f(n)$ where $\theta(n)$ is (completely) multiplicative and f(n) is (completely) additive. We then prove that if F(n) is distributive so is $\overline{F}(n) = \sum_{d|n} F(d)$ and conversely. Further, if F is distributive so is $F'(n) = \sum_{d|n} F(d)\mu(n/d)$. Finally, we prove that if F(n) is distributive so is $F^*(n) = \sum_{d|n} F(d)h(d)$, where h(n) is any multiplicative function and in particular $\sum_{d|n} F(d)\mu(d)$ is also distributive. We conclude the paper by noting that the set of all distributive functions with values in a field is a group isomorphic to the direct product of the groups of multiplicative and additive functions with values in the same field. (Received September 18, 1972.)

*701-10-2. DONALD G. HAZLEWOOD, Southwest Texas State University, San Marcos, Texas 78666. Sums over positive integers with few prime factors.

The author presents estimates for sums of multiplicative functions satisfying certain conditions extended over positive integers n such that n is less than or equal to x^t, the greatest prime factor of n is at most equal to x, and n is relatively prime to a natural number k, extending three of the major theorems by B. V. Levin and A. S. Fainleib, "Application of some integral equations to problems in number theory," Uspehi Mat. Nauk 22 (1967), 119-197 (=Russian Math. Surveys 22(1967), 119-204), to estimates uniform in t, x, and k. As a specific application, the author proves an estimate for $\Psi_k(x^t, x)$, the number of positive integers n such that n is less than or equal to x^t, the greatest prime factor of n is at most equal to x, and n is relatively prime to k, that is uniform in t, x, and k. (Received September 22, 1972.)

*701-10-3. GERALD B. HUFF, University of Georgia, Athens, Georgia 30601. On pairings of the first 2n natural numbers.

An affirmative, constructive answer is given to the conjecture of Mok-Kong Shen and Tsen-Pao Shen (Bull. Amer. Math. Soc. 68(1962), 557) that for $n > 2$, there exists a pairing, $\{(a_i, b_i) \mid a_i < b_i\}$, of the first 2n natural numbers such that the 2n numbers $b_i + a_i$ and $b_i - a_i$ are all different. Indeed, it is shown that for n relatively prime to 6, there is a unique pairing of the first 2n natural numbers satisfying the above conditions and $(a_i, b_i) \equiv (i, 2i)$ modulo n. These pairings for $n = 6k \pm 1$ are modified slightly to provide solutions of the Shen problem for $n = 6k - 2$, 6k, $6k + 2$, and $6k + 3$. (Received October 5, 1972.)