An Introduction to Maple - Part 2

In this session, you will explore how one can work with functions in Maple.
We will discuss some basic commands that can be used to define, to manipulate, and to plot functions.

Expressions

A math expression is a sensible string of symbols.

```
> expr1:=(x^2+2*a*y-1)/cos(x);
```

Expressions can be manipulated in ways you are familiar with. You can add, subtract, multiply, and divide expressions to produce new expressions. You can substitute one expression into another. You can have Maple rearrange the terms of an expression to display it in a more convenient form.

```
> expr2:=(2*x^2+1)/(x^2-1);
> expr3:=(x*y-3*y+1)/(x+1);
> S:=expr2+expr3;
> expr4:=subs(y=u+v^2,S);
```

The command 'subs' is the substitution command. We substituted the expression $u + v^2$ for y in expression S at each place y occurred. The previous expression (*expr4*) is a sum of two rational expressions. We can convert this sum to normal form with the 'normal' command. This command finds a common denominator, combines rational expressions, and eliminates common factors of the numerator and the denominator.

```
> N:=normal(S);
> collect(N,x);
```

The 'collect' command collects coefficients of like powers of a variable in a polynomial. In the expression N we can collect coefficients of x, u, or v.

```
> collect(N,u);
> collect(N,v);
> collect(N,z);
```

Nothing happened! When we collected with respect to z Maple simply printed the expression N without collecting.

Functions

Functions versus Expressions

Maple recognizes the difference between functions and expressions. Expressions are passive strings of symbols. Functions involve ''action'': they accept input values, perform some calculation, and produce an output. It is easy to blur over the distinction, because we often use expressions to define functions. But we will find that Maple handles expressions and functions differently, and we will have to learn the differences to make sure we program Maple correctly.

The correct way to define a function is with the arrow symbol.

```
> F:= x -> 3*x^2+2*x-1;
```

Let's define the corresponding expression and compare the two.

```
> e:= 3*x^2+2*x-1;
```

Open a new worksheet, and compare how Maple handles functions and expressions by evaluating the following: $\mathrm{F}(2)$, $\mathrm{e}(2)$; $\mathrm{subs}(x=2, F)$, $\mathrm{subs}(x=2, e)$; $\mathrm{factor}(F)$, $\mathrm{factor}(e)$. The difference between expressions and functions will become clearer as you gain experience with Maple.

Maple has ways to convert an expression to a function and a function to an expression. To convert our function F to an expression, we simply write $\mathrm{F}(x)$. The result is the output expected when we input the symbol x into the function F.

```
> f:=F(x);
> e-f;
```

To convert from an expression to a function, we use the 'unapply' command. Let's convert the expression e (from above) to a function E. As you might expect, the function E is the same as the function F (from above).

```
> E:=unapply(e,x);
```

Plotting Functions

Let's define some functions.

```
> restart;
> F:= x -> 3*x^2+2*x-1;
> G:= x -> 3*sin(x)+1/x;
```

The plotting command is 'plot', and in its simplest form we only need to tell Maple the name of the function we want to plot and the plotting interval.

```
> plot(F(x),x=-2..2);
> plot(G(x),x=-2..2);
> plot(G(x),x=-2..2,y=-10..10,discont=true);
> plot([F(x),G(x),F(x)-G(x)],x=-2..2,y=-10..10,color=[red,blue,green],discont=true);
```

Maple handles piecewise-defined function fairly well.

```
> H:= x -> piecewise(x<-1,-1, x<1, 2,6);
> J:= H+sin;
> simplify(J(x));
> plot(J(x),x=-5..5,discont=true,color=red);
> K:= x -> piecewise(x<-2,x,x<1,10-4*x-5*x^2,x^4);
> simplify(K(x));
> plot(K(x),x=-3..2);
> simplify((J+K)(x));
> plot((J+K)(x),x=-3..2,discont=true,color=red);
> simplify((J@K)(x));
> plot((J@K)(x),x=-4..3,color=red);
```

Practice Exercises

Let's practice what we have learned.

1. Plot the following functions: $y = \sin(1 - \sqrt{x})$, $y = 1 - \sqrt{\sin(x)}$, $y = \left|\,|2\,x| - |x+1|\,\right|$.
(To obtain a correct view of the second graph you may want to set the option $discont = true$.)

2. Define the functions $\mathrm{F}(x) = \mathbf{e}^x$ and $\mathrm{G}(x) = 20\sin(x)$. Plot the functions F, G, and $F + G$ on the same coordinate system over the interval $[-4, 4]$.

3. Let $H = F + G$, where F and G are from the previous problem, and plot the graphs of:
(a) H;
(b) H shifted one unit to the right;
(c) H reflected about the y axis;
(d) H compressed horizontally by a factor of two, compressed vertically by two, shifted left one unit, and shifted down three units.
(Choose the appropriate viewing rectangle.)

4. By shrinking the viewing rectangle appropriately, try to estimate the first positive x where the graph of $y = \tan(x)$ intersects the graph of $y = x$. Estimate x to three digits of accuracy. Afterwards, apply *fsolve* to the equation $\tan(x) = x$ to confirm your estimate.