

FOUNDATIONS OF MATH (MATH 4000/5000)

REVIEW SHEET I

I. Sentential logic/Propositional logic

- (a) The alphabet of symbols for sentential logic.
- (b) Sentential formulas.
- (c) Formula trees.
- (d) Unique readability of formulas.
- (e) Truth assignments.
- (f) Tautologies and contradictions.
- (g) Completeness of sets of connectives.
- (h) Disjunctive normal form.
- (i) Truth ($\models$, valuations) versus provability ($\vdash$, proofs, modus ponens).
- (j) Soundness, completeness, and decidability of a proof system.
- (k) The Compactness Theorem for sentential logic.
- (l) The proof that Compactness implies Completeness.

II. First-order logic

- (a) The alphabet of symbols for first-order logic.
- (b) Terms, atomic formulas, first-order formulas.
- (c) Formula trees.
- (d) Structures.
- (e) Valuations in a structures. Valid formulas.
- (f) Truth ($\models$, valuations).
- (g) Definability of a class of structures by sentences.
- (h) Elementary class, elementary equivalence, theory.
- (i) Definability within a structure by formulas. Expanding the language by definable relations.
- (j) The use of automorphisms to establish undefinability within a structure.

General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

- (i) Know the definitions of new concepts, and the meanings of the definitions.
- (ii) Know the statements and meanings of the major theorems.
- (iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
- (iv) Know how to perform the different kinds of calculations discussed in class.
- (v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
- (vi) Know how to correct mistakes made on old HW.