Foundations of Mathematics
Quiz 9

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. When appropriate, show your work and give adequate explanations.

1. Create one reasonable quiz question (that you know how to answer) based on the material from last week:

Nov 2	We reviewed a strategy for proving the Completeness Theorem. One new element that we discussed was the property of a complete, consistent theory having witnesses. Quiz 8 solutions.
Nov 4	We reviewed Quiz 8. We began a proof of the Completeness Theorem following these slides.
Nov 6	We discussed the metatheorems supporting the first half of the proof of the Completeness Theorem. Near the end of class we worked in groups on Exercise 2.4(9) from the textbook.

The questions posed (slightly paraphrased, one question per paper) were:

(1) Explain why, if we can prove $P(c)$ where c is not in the assumptions, we can deduce $(\forall x)P(x)$.
(2) Why are the following equivalent? "$\Gamma \vDash \alpha$ implies $\Gamma \vdash \alpha$" and "If Γ is consistent, then Γ is satisfiable".
(3) Give an example where $(\varphi_y^x)_x^y$ does not equal φ.
(4) What does it mean for a set of formulas to have witnesses? (Asked twice in slightly different forms.)
(5) Let Γ be a consistent set of formulas and let E be a larger set of formulas. If, for each $\alpha \in E \setminus \Gamma$, the set $\Gamma \cup \{\alpha\}$ is consistent, is E necesssarily consistent?
(6) Why must Γ be extended in order to prove the Completeness Theorem? Why do we need witnesses?
(7) Show that if Γ is satisfiable and consistent, then any subset of Γ is satisfiable and consistent.
(8) Show that you can expand a consistent set of formulas Γ to a set Δ such that for all formulas α either $\alpha \in \Delta$ or $\neg\alpha \in \Delta$.
(9) What properties must Γ have to use the theorem that "$\Gamma \vDash \alpha$ implies $\Gamma \vdash \alpha$"?
(10) Define what it means for Γ' to be a maximal consistent set.
(11) If Γ_1 and Γ_2 are finite and satisfiable, must a maximal extension Δ for Γ_1 be the same as a maximal extension Δ for Γ_2?