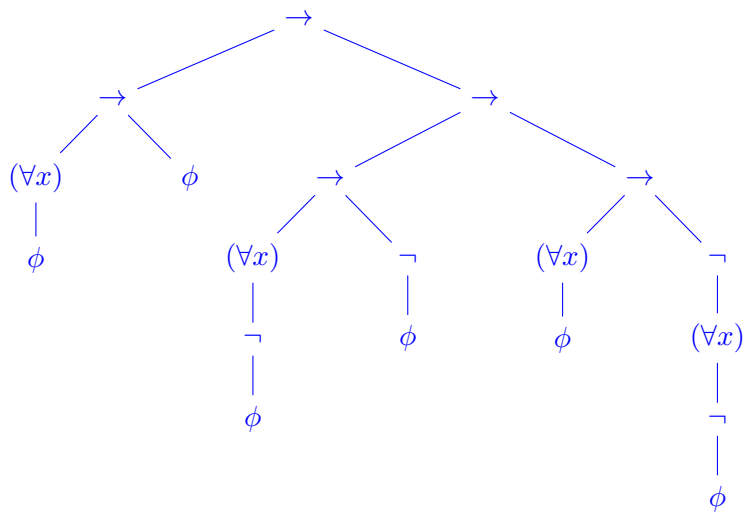
Foundations of Mathematics
Quiz 7

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. When appropriate, show your work and give adequate explanations.

1. Draw a formula tree for

$$((\forall x)\phi \to \phi) \to \Big(((\forall x)(\neg\phi) \to (\neg\phi)) \to ((\forall x)\phi \to \neg(\forall x)(\neg\phi))\Big).$$



Setting $p = (\forall x)\phi$, $q = \phi$, and $r = (\forall x)\neg\phi$ this reduces to

$$(\bigstar) \qquad (p \to q) \to ((r \to \neg q) \to (p \to \neg r)).$$

That is, the formula of Problem 1 is a substitution instance of ($\bigstar$).

2. Show that the formula in Problem 1 is a tautology.

To show that ($\bigstar$) is a propositional tautology, try to locate a row of the truth table for ($\bigstar$) that makes it false. Necessarily, $(p \to q) = T$, $(r \to \neg q) = T$, and $(p \to \neg r) = F$. Examining these in reverse order leads to (i) $p = T, r = T$, (ii) $q = F$, and (iii) $p = F$. Thus, $T = p = F$, a contradiction. That is, there is no row of the truth table for ($\bigstar$) where it is false.