

Foundations of Mathematics
Quiz 3

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete, legible** and **correct**. When appropriate, show your work and give adequate explanations.

1. State the Compactness Theorem for Propositional Logic.

A set of formulas is satisfiable iff it is finitely satisfiable.

2. Assume that $\Gamma \cup \{\alpha, \beta\}$ is a set of propositional formulas. Show that if $\{\alpha, \alpha \rightarrow \beta, \neg\beta\} \subseteq \Gamma$, then Γ is not finitely satisfiable.

It suffices to show that the finite subset $\{\alpha, \alpha \rightarrow \beta, \neg\beta\}$ of Γ is not satisfiable.

Suppose to the contrary that there is a satisfying valuation $\bar{v}$ such that

- (1) $\bar{v}(\alpha) = 1$,
- (2) $\bar{v}(\alpha \rightarrow \beta) = 1$,
- (3) $\bar{v}(\neg\beta) = 1$.

Item (3), together with the fact that valuations respect $\neg$, implies $\bar{v}(\beta) = \neg 1 = 0$. This fact + Item (1) + the fact that valuations respect $\rightarrow$ jointly imply that $\bar{v}(\alpha \rightarrow \beta) = 0$, contrary to Item (2).