Foundations of Mathematics
Quiz 10

Name:_____

You have 10 minutes to complete this quiz. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. When appropriate, show your work and give adequate explanations.

1. The Completeness Theorem is proved by showing that certain sets of L-formulas are satisfiable. The sets Δ used in the proof of the theorem have the properties

(1) Δ is consistent,
(2) for every L-formula α, either $\alpha \in \Delta$ or $\neg\alpha \in \Delta$, and
(3) Δ has witnesses.

Describe how to create the satisfying structure $\mathbf{A} = \langle A; C, F, P\rangle$ that is used in the proof. (This means: explain the choices for A = universe, and the interpretations of C=set of constants in the language, F=set of function symbols in the language, and P=set of predicates in the language.)

In a language without equality, we define $\mathbf{A} = \langle A; C, F, P\rangle$ by choosing

(1) A = set of terms in the language.
(2) $c^{\mathbf{A}} = c$ for each constant symbol $c \in C$.
(3) $f^{\mathbf{A}}(t_1, \ldots, t_m) = f(t_1, \ldots, t_m)$ for each function symbol $f \in F$.
(4) $p^{\mathbf{A}}(t_1, \ldots, t_n) = \top$ iff $p(t_1, \ldots, t_n) \in \Delta$ for each predicate symbol $p \in P$.

In a language with equality, we instead use the quotient of the previous structure by the following equivalence relation:

$$E = \{(s, t) \in A \times A \mid (s = t) \in \Delta\}.$$