

Induction proofs: examples and nonexamples

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Exercise. Prove $1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$ in a similar way.

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Inductive Step. Assume that $n \geq 4$ and that $1, 2, \dots, n$ are all products of primes.

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$$N_k = \binom{k}{4} + \binom{k-1}{2} + \binom{k}{1} = \frac{k^4 - 6k^3 + 23k^2 - 18k + 24}{24}.$$

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Proof. List finitely many horses as $(h_0, h_1, h_2, \dots, h_n)$. The claim is S_n : in any such list involving n horses, all h_i in the list have the same color.

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Corollary.

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Corollary. All horses are white.

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Mistaken induction proofs, 4

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Theorem. π is rational.

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We only proved that all finite decimal approximations π are rational, but not that the limit π is rational.