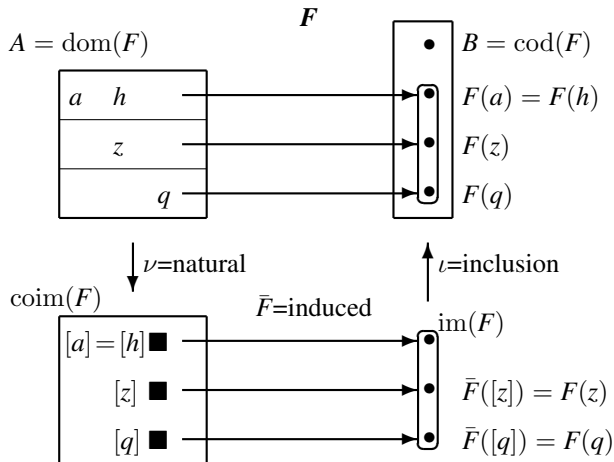


Functions: vocabulary and notation

Functions

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Exercise. Give examples and nonexamples exhibiting proper/improper use of the concept or notation.

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Definition. A **partition** of A is a set P consisting of nonempty subsets of A satisfying

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