

DISCRETE MATH

MIDTERM

You have 50 minutes for this exam. You may not use any unauthorized sources, and you may not communicate with others about the exam. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

I have neither given nor received aid on this exam.

Name: _____

(1) Fill in the blank with all appropriate choices (if any exist).

(a) Naive set theory is inconsistent.

(i) free of any contradictions.

(ii) inconsistent.

(iii) called ZFC. (Zermelo-Fraenkel set theory with the Axiom of Choice.)

(b) The sentence

$$\forall x \forall y \exists p \forall z ((z \in p) \leftrightarrow (z = x \text{ or } z = y))$$

expresses the Axiom of Pairing.

(i) Extensionality.

(ii) Power Set.

(iii) Pairing.

(2) (a) Define **the set of natural numbers**.

The set of natural numbers is the intersection of all inductive sets.

(b) Define what it means to say **the set X is finite**.

X is finite if there is a bijection $f: k \rightarrow X$ for some $k \in \mathbb{N}$.

(3) Give an example, if one exists, of each of the following. If no example exists, say why.

(a) A function $f: A \rightarrow B$ where $|\text{coim}(f)| \neq |\text{im}(f)|$.

No example exists, because the induced map $\bar{f}: \text{coim}(f) \rightarrow \text{im}(f)$ is a bijection.

(b) A partition of $\mathbb{N}$ with exactly two cells.

$\{\{0\}, \{n \in \mathbb{N} \mid n \neq 0\}\}$ is a partition of $\mathbb{N}$ into exactly two cells: the singleton set containing only 0 and the set of nonzero natural numbers.

(4) Let f be the function from $\{0, 1, 2\}$ to $\{a, b, c\}$ defined by $f(0) = f(1) = a$ and $f(2) = b$.

(a) Write $\text{coim}(f)$ as a set.

$$\{\{0, 1\}, \{2\}\}.$$

(b) Write $\ker(f)$ as a set.

$$\{(0, 0), (1, 1), (2, 2), (0, 1), (1, 0)\}$$

(5) Give the recursive definition of addition of natural numbers. Use it to show that $S(m) = m + 1$.

$$\begin{array}{ll} m + 0 & := m & \text{(IC)} \\ m + S(n) & := S(m + n) & \text{(RR)} \end{array}$$

For the second statement, for any m we have

$$\begin{array}{ll} m + 1 & = m + S(0) & \text{(Definition of number 1)} \\ & = S(m + 0) & \text{(RR, +)} \\ & = S(m) & \text{(IC, +)} \end{array}$$