

Discrete Math
Quiz 4

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

Define the factorial function $n!$ recursively, as follows:

$$\begin{aligned} 0! &= 1 \\ (n+1)! &= (n+1) \cdot n! \end{aligned}$$

1. Use induction to prove that if $n \in \mathbb{N}$, then $n! \neq 0$.

Base Case. ($n = 0$)

If $n = 0$, then $n! = 1$ by the Initial Condition of the recursive definition of the factorial function. We have $1 \neq 0$, since $1 = S(0)$ is a successor and 0 is not a successor. Hence $0! = 1 \neq 0$.

Inductive Step. (Assume that $n! \neq 0$ and prove that $(n+1)! \neq 0$)

By the Recursive Rule of the definition of the factorial function, $(n+1)! = (n+1) \cdot n!$. We have $n+1 \neq 0$, since $n+1$ is a successor and 0 is not a successor. We have $n! \neq 0$ by the Inductive Hypothesis. Thus, $(n+1)! = (n+1) \cdot n!$ is the product of two nonzero natural numbers. This implies $(n+1)! \neq 0$ by the “Zero-Product Property” (or “no Zero Divisors Property”) of multiplication of natural numbers.