

Consequence

Some Definitions

Some Definitions

① $\mathbf{A} \models \alpha$

Some Definitions

① $\mathbf{A} \models \alpha$

Some Definitions

① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$)

Some Definitions

① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:

Some Definitions

- 1 $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α

Some Definitions

- 1 $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
 $\mathbf{A}$ satisfies α .

Some Definitions

- 1 $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
 $\mathbf{A}$ satisfies α .

Example.

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$,

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example.

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).

A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .

Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).

A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .

Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$ means:

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$,

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
 $\mathbf{A}$ satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, $\alpha[v]$ is true.
 α is a valid sentence.

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
A satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, $\alpha[v]$ is true.
 α is a valid sentence.

Example.

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
 $\mathbf{A}$ satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, $\alpha[v]$ is true.
 α is a valid sentence.

Example. $\models ((\exists x)(\forall y)(x \leq y) \rightarrow (\forall y)(\exists x)(x \leq y))$.

Some Definitions

- ① $\mathbf{A} \models \alpha$ (or $\models_{\mathbf{A}} \alpha$) means:
for every valuation v in $\mathbf{A}$, v satisfies α ($\alpha[v]$ is true).
 $\mathbf{A}$ satisfies α .

Example. $\mathbb{N} \models (x + y) = (y + x)$.

- ② $\Gamma \models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, if v satisfies all the formulas in Γ , then v also satisfies α .
 Γ entails α .

Example. $\{xy = yx, x(yz) = (xy)z\} \models x(yz) = z(xy)$.

- ③ $\models \alpha$ means:
for every L -structure $\mathbf{A}$ and every valuation v in $\mathbf{A}$, $\alpha[v]$ is true.
 α is a valid sentence.

Example. $\models ((\exists x)(\forall y)(x \leq y) \rightarrow (\forall y)(\exists x)(x \leq y))$.

Failures of those Definitions

Failures of those Definitions

① $\mathbf{A} \not\models \alpha$

Failures of those Definitions

① $\mathbf{A} \not\models \alpha$

Failures of those Definitions

① $A \not\models \alpha$ or $(\not\models_A \alpha)$

Failures of those Definitions

① $A \not\models \alpha$ or $(\not\models_A \alpha)$ means:

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
 $\mathbf{A}$ does not satisfy α .

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$ means:

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
A does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that $\alpha[v]$ is false.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
 $\mathbf{A}$ does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that $\alpha[v]$ is false.
 α is not a valid sentence.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
 $\mathbf{A}$ does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that $\alpha[v]$ is false.
 α is not a valid sentence.

Example.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
 $\mathbf{A}$ does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that $\alpha[v]$ is false.
 α is not a valid sentence.

Example. $\not\models ((\forall x)(\forall y)(P(x, y) \rightarrow P(y, x)))$.

Failures of those Definitions

- ① $\mathbf{A} \not\models \alpha$ or $(\not\models_{\mathbf{A}} \alpha)$ means:
there exists a valuation v in $\mathbf{A}$ that does not satisfy α ($\alpha[v]$ is false).
 $\mathbf{A}$ does not satisfy α .

Example. $\mathbb{N} \not\models ((x + x) = x)$.

- ② $\Gamma \not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that v satisfies all the formulas in Γ but v does not satisfy α .
 Γ does not entail α .

Example. $\{xy = yx, x(yz) = (xy)z\} \not\models ((xx) = x)$.

- ③ $\not\models \alpha$ means:
there exists an L -structure $\mathbf{A}$ and a valuation v in $\mathbf{A}$ such that $\alpha[v]$ is false.
 α is not a valid sentence.

Example. $\not\models ((\forall x)(\forall y)(P(x, y) \rightarrow P(y, x)))$.

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

Let $\mathbf{A}$ be a structure.

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

Let $\mathbf{A}$ be a structure.

Let $v_1, v_2: V \rightarrow A$ be two valuations.

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

Let $\mathbf{A}$ be a structure.

Let $v_1, v_2: V \rightarrow A$ be two valuations.

Let $I \subseteq \mathbb{N}$ be a subset.

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

Let $\mathbf{A}$ be a structure.

Let $v_1, v_2: V \rightarrow A$ be two valuations.

Let $I \subseteq \mathbb{N}$ be a subset.

Say that v_1 and v_2 are I -**equivalent** if $v_1(x_i) = v_2(x_i)$ for all $i \in I$.

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

Let $\mathbf{A}$ be a structure.

Let $v_1, v_2: V \rightarrow A$ be two valuations.

Let $I \subseteq \mathbb{N}$ be a subset.

Say that v_1 and v_2 are I -**equivalent** if $v_1(x_i) = v_2(x_i)$ for all $i \in I$. (The two valuations agree on the variables whose subscripts belong to I .)

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

Let $\mathbf{A}$ be a structure.

Let $v_1, v_2: V \rightarrow A$ be two valuations.

Let $I \subseteq \mathbb{N}$ be a subset.

Say that v_1 and v_2 are I -**equivalent** if $v_1(x_i) = v_2(x_i)$ for all $i \in I$. (The two valuations agree on the variables whose subscripts belong to I .)

Say that v_1 and v_2 are $\bar{I}$ -**equivalent** if $v_1(x_i) = v_2(x_i)$ for all $i \notin I$.

I -equivalent valuations

Let $V = \{x_i \mid i \in \mathbb{N}\}$ be the set of variables of a language.

Let $\mathbf{A}$ be a structure.

Let $v_1, v_2: V \rightarrow A$ be two valuations.

Let $I \subseteq \mathbb{N}$ be a subset.

Say that v_1 and v_2 are I -**equivalent** if $v_1(x_i) = v_2(x_i)$ for all $i \in I$. (The two valuations agree on the variables whose subscripts belong to I .)

Say that v_1 and v_2 are $\bar{I}$ -**equivalent** if $v_1(x_i) = v_2(x_i)$ for all $i \notin I$. (The two valuations agree on the variables whose subscripts do not belong to I .)

How valuations are used to determine whether $A \models \alpha$

How valuations are used to determine whether $A \models \alpha$

Stage 0.

How valuations are used to determine whether $A \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

How valuations are used to determine whether $A \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

Stage 1.

How valuations are used to determine whether $\mathbf{A} \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

Stage 1. Define $R(t_1, \dots, t_n)[v]$ to be True if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \in R^{\mathbf{A}}$ and False if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \notin R^{\mathbf{A}}$.

How valuations are used to determine whether $\mathbf{A} \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

Stage 1. Define $R(t_1, \dots, t_n)[v]$ to be True if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \in R^{\mathbf{A}}$ and False if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \notin R^{\mathbf{A}}$.

Stage 2.

How valuations are used to determine whether $\mathbf{A} \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

Stage 1. Define $R(t_1, \dots, t_n)[v]$ to be True if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \in R^{\mathbf{A}}$ and False if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \notin R^{\mathbf{A}}$.

Stage 2. Define $(\neg\alpha)[v]$ to be the negation of $\alpha[v]$.

How valuations are used to determine whether $\mathbf{A} \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

Stage 1. Define $R(t_1, \dots, t_n)[v]$ to be True if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \in R^{\mathbf{A}}$ and False if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \notin R^{\mathbf{A}}$.

Stage 2. Define $(\neg\alpha)[v]$ to be the negation of $\alpha[v]$. Define $(\alpha \rightarrow \beta)[v]$ to be True iff it is not the case that $\alpha[v] = T$ and $\beta[v] = F$.

How valuations are used to determine whether $\mathbf{A} \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

Stage 1. Define $R(t_1, \dots, t_n)[v]$ to be True if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \in R^{\mathbf{A}}$ and False if $(t_1^{\mathbf{A}}[v], \dots, t_n^{\mathbf{A}}[v]) \notin R^{\mathbf{A}}$.

Stage 2. Define $(\neg\alpha)[v]$ to be the negation of $\alpha[v]$. Define $(\alpha \rightarrow \beta)[v]$ to be True iff it is not the case that $\alpha[v] = T$ and $\beta[v] = F$.

Stage 3.

How valuations are used to determine whether $A \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

Stage 1. Define $R(t_1, \dots, t_n)[v]$ to be True if $(t_1^A[v], \dots, t_n^A[v]) \in R^A$ and False if $(t_1^A[v], \dots, t_n^A[v]) \notin R^A$.

Stage 2. Define $(\neg\alpha)[v]$ to be the negation of $\alpha[v]$. Define $(\alpha \rightarrow \beta)[v]$ to be True iff it is not the case that $\alpha[v] = T$ and $\beta[v] = F$.

Stage 3. Define $(\forall x_i)\alpha[v]$ to be True iff $\alpha[v']$ is True for all v' that is $\overline{\{i\}}$ -equivalent to v .

How valuations are used to determine whether $A \models \alpha$

Stage 0. Recursively extend the definition of v : Variables $\rightarrow A$ to v : Terms $\rightarrow A$.

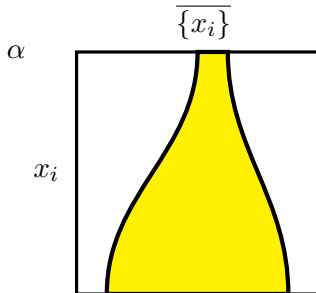
Stage 1. Define $R(t_1, \dots, t_n)[v]$ to be True if $(t_1^A[v], \dots, t_n^A[v]) \in R^A$ and False if $(t_1^A[v], \dots, t_n^A[v]) \notin R^A$.

Stage 2. Define $(\neg\alpha)[v]$ to be the negation of $\alpha[v]$. Define $(\alpha \rightarrow \beta)[v]$ to be True iff it is not the case that $\alpha[v] = T$ and $\beta[v] = F$.

Stage 3. Define $(\forall x_i)\alpha[v]$ to be True iff $\alpha[v']$ is True for all v' that is $\overline{\{i\}}$ -equivalent to v . Define $(\exists x_i)\alpha[v]$ to be True iff $\alpha[v']$ is True for some v' that is $\overline{\{i\}}$ -equivalent to v .

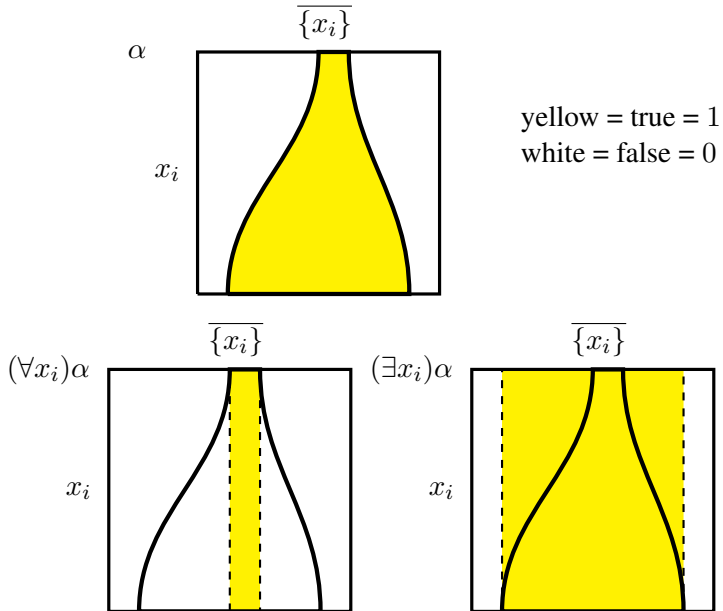
The effect of quantification on tables

The effect of quantification on tables



yellow = true = 1
white = false = 0

The effect of quantification on tables



Some Properties

Some Properties

- ① If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true.

Some Properties

- ① If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true.

Some Properties

- ① If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α .)

Some Properties

- ① If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)

Some Properties

- ① If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)
- ② If x_i does not appear free in α , and $v, v': V \rightarrow A$ are valuations that are $\overline{\{i\}}$ -equivalent, then $\alpha[v]$ is true in $\mathbf{A}$ if and only if $\alpha[v']$ is true in $\mathbf{A}$.

Some Properties

- ① If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)
- ② If x_i does not appear free in α , and $v, v': V \rightarrow A$ are valuations that are $\overline{\{i\}}$ -equivalent, then $\alpha[v]$ is true in $\mathbf{A}$ if and only if $\alpha[v']$ is true in $\mathbf{A}$.

Some Properties

- 1 If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)
- 2 If x_i does not appear free in α , and $v, v': V \rightarrow A$ are valuations that are $\overline{\{i\}}$ -equivalent, then $\alpha[v]$ is true in $\mathbf{A}$ if and only if $\alpha[v']$ is true in $\mathbf{A}$.
- 3 If α is a sentence

Some Properties

- 1 If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)
- 2 If x_i does not appear free in α , and $v, v': V \rightarrow A$ are valuations that are $\overline{\{i\}}$ -equivalent, then $\alpha[v]$ is true in $\mathbf{A}$ if and only if $\alpha[v']$ is true in $\mathbf{A}$.
- 3 If α is a sentence

Some Properties

- 1 If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)
- 2 If x_i does not appear free in α , and $v, v': V \rightarrow A$ are valuations that are $\overline{\{i\}}$ -equivalent, then $\alpha[v]$ is true in $\mathbf{A}$ if and only if $\alpha[v']$ is true in $\mathbf{A}$.
- 3 If α is a sentence (a formula with no free variables),

Some Properties

- 1 If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)
- 2 If x_i does not appear free in α , and $v, v': V \rightarrow A$ are valuations that are $\overline{\{i\}}$ -equivalent, then $\alpha[v]$ is true in $\mathbf{A}$ if and only if $\alpha[v']$ is true in $\mathbf{A}$.
- 3 If α is a sentence (a formula with no free variables), then any two valuations assign the same truth value to α in $\mathbf{A}$.

Some Properties

- ① If the set of variables appearing in the formula α have subscripts contained in the set $I \subseteq \mathbb{N}$, and v and v' are I -equivalent valuations in $\mathbf{A}$, then $\alpha[v]$ is true iff $\alpha[v']$ is true. (This can be proved by Induction on the complexity of α . See Theorem 22A.)
- ② If x_i does not appear free in α , and $v, v': V \rightarrow A$ are valuations that are $\overline{\{i\}}$ -equivalent, then $\alpha[v]$ is true in $\mathbf{A}$ if and only if $\alpha[v']$ is true in $\mathbf{A}$.
- ③ If α is a sentence (a formula with no free variables), then any two valuations assign the same truth value to α in $\mathbf{A}$. (Corollary 22B.)

Dependence on a variable

Dependence on a variable

Definition.

Dependence on a variable

Definition. If α is a formula and x_i is a variable , then the meaning of α **does not depend on** x_i

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

- ① $(x_1 \cdot 0 = 0)$ does not depend on x_1 in $\mathbb{R}$.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

- ① $(x_1 \cdot 0 = 0)$ does not depend on x_1 in $\mathbb{R}$.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

- ① $(x_1 \cdot 0 = 0)$ does not depend on x_1 in $\mathbb{R}$.
- ② $(x_1 \cdot 0 = 0)$ does not depend on x_2 in $\mathbb{R}$.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

- ① $(x_1 \cdot 0 = 0)$ does not depend on x_1 in $\mathbb{R}$.
- ② $(x_1 \cdot 0 = 0)$ does not depend on x_2 in $\mathbb{R}$.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

- ① $(x_1 \cdot 0 = 0)$ does not depend on x_1 in $\mathbb{R}$.
- ② $(x_1 \cdot 0 = 0)$ does not depend on x_2 in $\mathbb{R}$.
- ③ $(\forall x_1)(x_1 \cdot x_1 = x_1)$ does not depend on x_1 in $\mathbb{R}$.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

- ① $(x_1 \cdot 0 = 0)$ does not depend on x_1 in $\mathbb{R}$.
- ② $(x_1 \cdot 0 = 0)$ does not depend on x_2 in $\mathbb{R}$.
- ③ $(\forall x_1)(x_1 \cdot x_1 = x_1)$ does not depend on x_1 in $\mathbb{R}$.

Dependence on a variable

Definition. If α is a formula and x_i is a variable, then the meaning of α **does not depend on** x_i in $\mathbf{A}$ if for all $\overline{\{i\}}$ -equivalent valuations v, v' in $\mathbf{A}$ we have that $\alpha[v] = T$ iff $\alpha[v'] = T$.

Examples.

- ① $(x_1 \cdot 0 = 0)$ does not depend on x_1 in $\mathbb{R}$.
- ② $(x_1 \cdot 0 = 0)$ does not depend on x_2 in $\mathbb{R}$.
- ③ $(\forall x_1)(x_1 \cdot x_1 = x_1)$ does not depend on x_1 in $\mathbb{R}$.

Some logical equivalences

Some logical equivalences

Exercises.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.
 - 3 $(\alpha \wedge ((\forall x)\beta)) \models (\forall x_i)(\alpha \wedge \beta)$ when α does not depend on x_i .

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.
 - 3 $(\alpha \wedge ((\forall x)\beta)) \models (\forall x_i)(\alpha \wedge \beta)$ when α does not depend on x_i .

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.
 - 3 $(\alpha \wedge ((\forall x)\beta)) \models (\forall x_i)(\alpha \wedge \beta)$ when α does not depend on x_i .
 - 4 $(\alpha \vee ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \vee \beta)$ when α does not depend on x_i .

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.
 - 3 $(\alpha \wedge ((\forall x)\beta)) \models (\forall x_i)(\alpha \wedge \beta)$ when α does not depend on x_i .
 - 4 $(\alpha \vee ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \vee \beta)$ when α does not depend on x_i .

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.
 - 3 $(\alpha \wedge ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \wedge \beta)$ when α does not depend on x_i .
 - 4 $(\alpha \vee ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \vee \beta)$ when α does not depend on x_i .
 - 5 $(\alpha \rightarrow ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \rightarrow \beta)$ when α does not depend on x_i .

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.
 - 3 $(\alpha \wedge ((\forall x)\beta)) \models (\forall x_i)(\alpha \wedge \beta)$ when α does not depend on x_i .
 - 4 $(\alpha \vee ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \vee \beta)$ when α does not depend on x_i .
 - 5 $(\alpha \rightarrow ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \rightarrow \beta)$ when α does not depend on x_i .

Some logical equivalences

Exercises.

This exercise explores how quantifiers interact with logical connectives.

- 1 Explain the meaning of $\alpha \models \beta$.
- 2 Convince yourself that the following logical equivalences hold.
 - 1 $(\neg(\forall x_i)\alpha) \models (\exists x_i)(\neg\alpha)$.
 - 2 $(\neg(\exists x_i)\alpha) \models (\forall x_i)(\neg\alpha)$.
 - 3 $(\alpha \wedge ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \wedge \beta)$ when α does not depend on x_i .
 - 4 $(\alpha \vee ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \vee \beta)$ when α does not depend on x_i .
 - 5 $(\alpha \rightarrow ((\forall x_i)\beta)) \models (\forall x_i)(\alpha \rightarrow \beta)$ when α does not depend on x_i .
 - 6 $((\forall x_i)\alpha \rightarrow \beta) \models (\exists x_i)(\alpha \rightarrow \beta)$ when β does not depend on x_i .