HW 4.

In this assignment we work over ZF (without assuming the Axiom of Choice).

A set X is *Dedekind Finite* if every injective function $f\colon X \to X$ is surjective. Otherwise X is *Dedekind infinite*.

A set A is *Amorphous* if it is infinite and every subset of A is finite or cofinite.

1. (Kai Morton, Orlando Reyes)
   Show that X is Dedekind infinite iff there is an injective function $f\colon \omega \to X$.

2. (Nick Cooper, Ben Kitchen)
   (a) Show that an amorphous set is Dedekind finite.
   (b) Show that if A is amorphous, then $|A| < |A \times A|$.

3. (Jonathan Bayley, Khizar Pasha)
   Show that an amorphous set cannot be totally ordered. (Hence, it cannot be well-ordered.)

4. (Mattan Feldman, Kai Morton)
   Show that if A is amorphous, then $\mathcal{P}(A)$ is Dedekind finite.

5. (Everyone!)
   Suppose that A is an amorphous set. Explain how to construct from A an inductively ordered poset with no maximal element. (This shows directly that Zorn's Lemma must fail in the presence of an amorphous set.)