

### HW 3.

1. (Mattan Feldman, Orlando Reyes) Show that the following statement is equivalent over ZFC to CH:

*The real plane  $\mathbb{R}^2$  may be partitioned into two cells,  $\{A, B\}$ , in such a way that any horizontal line in  $\mathbb{R}^2$  has countable intersection with  $A$  and any vertical line in  $\mathbb{R}^2$  has countable intersection with  $B$ .*

2. (Nick Cooper, Khizar Pasha) Show that if  $\kappa$  and  $\lambda$  are infinite cardinals and  $\kappa < \lambda$ , then there is an infinite cardinal  $\mu$  such that  $\mu^\kappa < \mu^\lambda$ .

3. (Jonathan Bayley, Kai Morton) Show that the following statement is equivalent over ZFC to GCH:

If  $\kappa$  is any infinite cardinal, then  $\kappa^{\text{cf}(\kappa)} = \kappa^+$ .

4. (Orlando Reyes, Ben Kitchen) Let  $\mathbb{F}$  be a field of size  $\kappa$  and let  $V$  be an  $\mathbb{F}$ -vector space of infinite dimension  $\lambda$ .

- (a) Show that the dimension of the dual space  $V^*$  is  $\kappa^\lambda$ .
- (b) Use the Main Theorem of Cardinal Arithmetic to simplify  $\dim_{\mathbb{F}}(V^*)$  in the case where  $|\mathbb{F}| = \beth_{\omega_1}$  and  $\dim_{\mathbb{F}}(V) = \aleph_0$ .