HW 2.

1. (Nick Cooper, Mattan Feldman) Find all pairs (κ, λ) of cardinals such that the cardinal sum $\kappa +_{\mathrm{c}} \lambda$ agrees with the ordinal sum $\kappa +_{\mathrm{o}} \lambda$.

2. (Ben Kitchen, Kai Morton) Show that
   (a) $|\mathcal{P}(\aleph_0)| = 2^{\aleph_0}$.
   (b) Show that the ordered set $\langle \mathcal{P}(\aleph_0); \subseteq \rangle$ contains a chain of cardinality $2^{\aleph_0}$.
   (c) Show that the ordered set $\langle \mathcal{P}(\aleph_0); \subseteq \rangle$ contains an antichain of cardinality $2^{\aleph_0}$.

3. (Khizar Pasha, Orlando Reyes) Show that
   (a) The set of permutations of $\aleph_0$ has cardinality $2^{\aleph_0}$.
   (b) The set of equivalence relations on $\aleph_0$ has cardinality $2^{\aleph_0}$.
   (c) The set of total orders on $\aleph_0$ has cardinality $2^{\aleph_0}$.

4. (Jonathan Bayley, Mattan Feldman)
   (a) Suppose that κ is an infinite cardinal and α is a smaller ordinal. Show that the interval $[\alpha, \kappa)$ of ordinals is order-isomorphic to κ.
   (b) Suppose that $\kappa_0 < \kappa_1 < \kappa_2 < \cdots$ is a strictly increasing sequence of cardinals with limit κ. Explain why each interval $[\kappa_i, \kappa_{i+1})$ is order-isomorphic to κ_{i+1}.
   (c) Suppose that $\kappa_0 < \kappa_1 < \kappa_2 < \cdots$ is an increasing sequence of cardinals with limit κ. Show that $\sum \kappa_i = \sup\{\kappa_i\} = \kappa$.