HW 1.

1. (Jonathan Bayley, Nick Cooper) Explain why each one of the following is a proper class.
   (a) The class $\mathcal{C}$ of all sets.
   (b) The class $\mathcal{D}$ of all 1-element sets.
   (c) The class $\mathcal{E}$ of all groups.
   (You should express both $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ as classes, and then that they are proper.)

2. (Mattan Feldman, Ben Kitchen) Let T be the theory axiomatized by all the axioms of ZFC except the Axiom of Foundation. ($T = \text{ZFC} \setminus \{\text{Fnd}\}$.) From T, prove that the Axiom of Foundation is equivalent to the following statement:
   There is no function f with domain ω such that $f(n+1) \in f(n)$ for all n.

3. (Kai Morton, Khizar Pasha) Explain why, if V is a model of ZFC, the natural numbers object in V will satisfy the first-order Peano axioms.

4. (Orlando Reyes, Jonathan Bayley) Prove that $m^{n+k} = m^n \cdot m^k$. (You may assume all the laws of <u>successor</u> and <u>addition</u> that we proved.)