

Meaning

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Example

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We will consider structures of the form $\langle A; +, \cdot, < \rangle$ and ask whether the following sentence is true:

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+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

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0	0	0	0
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2	0	2	1

$<$	0	1	2
0	<i>F</i>	<i>T</i>	<i>T</i>
1	<i>F</i>	<i>F</i>	<i>T</i>
2	<i>F</i>	<i>F</i>	<i>F</i>

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2	1	2	2

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2	1	2	2

$<$	0	1	2
0	F	T	T
1	F	F	T
2	F	F	F

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$<$	0	1	2
0	F	T	T
1	F	F	T
2	F	F	F

$t_1 < t_2$	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

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α	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

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α	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

 $\Rightarrow$

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

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α	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

 $\Rightarrow$

$(\exists y)\alpha$	0	1	2
0	F	F	F
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$(\exists y)\alpha$	0	1	2
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1	T	T	T
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 $\Rightarrow$

$(\forall x)(\exists y)\alpha$	0	1	2
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α	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

 $\Rightarrow$

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

 $\Rightarrow$

$(\forall x)(\exists y)\alpha$	0	1	2
0	F	F	F
1	F	F	F
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Conclusion:

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α	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

 $\Rightarrow$

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
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 $\Rightarrow$

$(\forall x)(\exists y)\alpha$	0	1	2
0	F	F	F
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Conclusion: The sentence is false in **A**.

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α	0	1	2
0	F	F	F
1	F	T	T
2	F	T	T

 $\Rightarrow$

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

$(\exists y)\alpha$	0	1	2
0	F	F	F
1	T	T	T
2	T	T	T

 $\Rightarrow$

$(\forall x)(\exists y)\alpha$	0	1	2
0	F	F	F
1	F	F	F
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Conclusion: The sentence is false in $\mathbf{A}$. ($\mathbf{A} \not\models \sigma$.)