Absoluteness

The concept of absoluteness, NST Chapter 13

The concept of absoluteness, NST Chapter 13

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V.

The concept of absoluteness, NST Chapter 13

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question:

The concept of absoluteness, NST Chapter 13

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question: Does the meaning of a formula change as we pass from M to N?

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question: Does the meaning of a formula change as we pass from M to N? For example, do "$x \in^M y$",

The concept of absoluteness, NST Chapter 13

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question: Does the meaning of a formula change as we pass from M to N? For example, do "$x \in^M y$", "$x \in^N y$", Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question: Does the meaning of a formula change as we pass from M to N? For example, do "$x \in^M y$", "$x \in^N y$", and "$x \in^V y$"

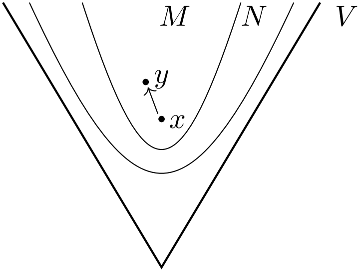
The concept of absoluteness, NST Chapter 13

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question: Does the meaning of a formula change as we pass from M to N? For example, do "$x \in^M y$", "$x \in^N y$", and "$x \in^V y$" mean the same thing for $x, y \in M$?

The concept of absoluteness, NST Chapter 13

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question: Does the meaning of a formula change as we pass from M to N? For example, do "$x \in^M y$", "$x \in^N y$", and "$x \in^V y$" mean the same thing for $x, y \in M$?

Let V be a model of ZF and let $M \subseteq N$ be transitive classes in V. We are concerned with the question: Does the meaning of a formula change as we pass from M to N? For example, do "$x \in^M y$", "$x \in^N y$", and "$x \in^V y$" mean the same thing for $x, y \in M$?



Relativization

Relativization

Let V be a model of ZF.

Relativization

Let V be a model of ZF. Let M be a class in V.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.)

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg\alpha)^M$ is $\neg\alpha^M$.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in\rangle$ is an induced substructure of $\langle V; \in\rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg\alpha)^M$ is $\neg\alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in\rangle$ is an induced substructure of $\langle V; \in\rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg\alpha)^M$ is $\neg\alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.
5. $((\forall x)\alpha)^M$ is $(\forall x)(\mu(x) \to \alpha^M)$.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg \alpha)^M$ is $\neg \alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.
5. $((\forall x)\alpha)^M$ is $(\forall x)(\mu(x) \to \alpha^M)$.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg \alpha)^M$ is $\neg \alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.
5. $((\forall x)\alpha)^M$ is $(\forall x)(\mu(x) \to \alpha^M)$.

Definition.

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in\rangle$ is an induced substructure of $\langle V; \in\rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg\alpha)^M$ is $\neg\alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.
5. $((\forall x)\alpha)^M$ is $(\forall x)(\mu(x) \to \alpha^M)$.

Definition. Let $M \subseteq N$ be classes in V.

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in\rangle$ is an induced substructure of $\langle V; \in\rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg\alpha)^M$ is $\neg\alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.
5. $((\forall x)\alpha)^M$ is $(\forall x)(\mu(x) \to \alpha^M)$.

Definition. Let $M \subseteq N$ be classes in V. φ is **absolute for** M, N if

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in \rangle$ is an induced substructure of $\langle V; \in \rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg\alpha)^M$ is $\neg\alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.
5. $((\forall x)\alpha)^M$ is $(\forall x)(\mu(x) \to \alpha^M)$.

Definition. Let $M \subseteq N$ be classes in V. φ is **absolute for** M, N if

$$(\forall \bar{x} \in M)(\varphi^M(\bar{x}) \leftrightarrow \varphi^N(\bar{x})).$$

Relativization

Let V be a model of ZF. Let M be a class in V. (Say, $x \in M$ iff $V \models \mu(x)$.) We assume that $\langle M; \in\rangle$ is an induced substructure of $\langle V; \in\rangle$. The **relativization** φ^M of a formula φ to M is defined by recursion:

1. $(x = y)^M$ is $(x = y)$.
2. $(x \in y)^M$ is $(x \in y)$.
3. $(\neg\alpha)^M$ is $\neg\alpha^M$.
4. $(\alpha \wedge \beta)^M$ is $\alpha^M \wedge \beta^M$.
5. $((\forall x)\alpha)^M$ is $(\forall x)(\mu(x) \to \alpha^M)$.

Definition. Let $M \subseteq N$ be classes in V. φ is **absolute for** M, N if

$$(\forall \bar{x} \in M)(\varphi^M(\bar{x}) \leftrightarrow \varphi^N(\bar{x})).$$

We say that φ is **absolute for** M if it is absolute for M, V.

Δ_0-formulas

Δ_0-formulas

Definition.

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification**

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all).

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$,

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples.

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n$ $(\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n\ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$
4. $x \times y$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$
4. $x \times y$
5. x is a binary relation

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$
4. $x \times y$
5. x is a binary relation
6. $\mathrm{dom}(x)$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$
4. $x \times y$
5. x is a binary relation
6. $\mathrm{dom}(x)$

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$
4. $x \times y$
5. x is a binary relation
6. $\mathrm{dom}(x)$

Exercises.

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$
4. $x \times y$
5. x is a binary relation
6. $\mathrm{dom}(x)$

Exercises. Verify for A(1), A(3), A(4), B(1), B(2).

Δ_0-formulas

Definition. A Δ_0-formula is one that involves only **bounded quantification** (possibly no quantification at all). "Bounded quantification" means quantification of the form $(\forall x \in y)$ or $(\exists x \in y)$, but does not include quantification of the form $(\forall x)$ or $(\exists x)$.

Examples. The following class functions and relations can be defined by Δ_0-formulas.

1. $x \in y$
2. $x = y$
3. $x \subseteq y$
4. $\{x, y\}$
5. $\{x\}$
6. (x, y)

1. $x = \emptyset$
2. $x \cup \{x\}$
3. $x = n \ (\in \omega)$
4. $x = \omega$
5. x is transitive
6. x is an ordinal

1. x is a limit ordinal
2. $x \cup y$
3. $x \cap y$
4. $x \times y$
5. x is a binary relation
6. $\mathrm{dom}(x)$

Exercises. Verify for A(1), A(3), A(4), B(1), B(2).

Example solutions

Example A(1).

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3).

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

$$(\forall z \in x)(z \in y)$$

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

$$(\forall z \in x)(z \in y)$$

Example B(5).

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

$$(\forall z \in x)(z \in y)$$

Example B(5). the relation "x is transitive" is expressible by the Δ_0-formula

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

$$(\forall z \in x)(z \in y)$$

Example B(5). the relation "x is transitive" is expressible by the Δ_0-formula

$$(\forall z \in x)((x \in y) \rightarrow (z \in y))$$

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

$$(\forall z \in x)(z \in y)$$

Example B(5). the relation "x is transitive" is expressible by the Δ_0-formula

$$(\forall z \in x)((x \in y) \to (z \in y))$$

Example B(6).

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

$$(\forall z \in x)(z \in y)$$

Example B(5). the relation "x is transitive" is expressible by the Δ_0-formula

$$(\forall z \in x)((x \in y) \to (z \in y))$$

Example B(6). the relation "x is an ordinal" is expressible by a Δ_0-formula that says "x is a transitive set of transitive sets":

Example solutions

Example A(1). "$x \in y$" is atomic, hence Δ_0.

Example A(3). the relation "$x \subseteq y$" is expressible by the Δ_0-formula

$$(\forall z \in x)(z \in y)$$

Example B(5). the relation "x is transitive" is expressible by the Δ_0-formula

$$(\forall z \in x)((x \in y) \to (z \in y))$$

Example B(6). the relation "x is an ordinal" is expressible by a Δ_0-formula that says "x is a transitive set of transitive sets":

$$\text{Transitive}(x) \wedge (\forall y \in x)\text{Transitive}(y)$$

Δ_0-Absoluteness

Theorem.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof:

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. $\checkmark$
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. $\checkmark$

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

For the third item, we need to justify the claim that if

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

For the third item, we need to justify the claim that if

$$(\forall xy\bar{z} \in M)(\alpha^M(x, y, \bar{z}) \leftrightarrow \alpha(x, y, \bar{z})),$$

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. $\checkmark$
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. $\checkmark$
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

For the third item, we need to justify the claim that if

$$(\forall x y \bar{z} \in M)(\alpha^M(x, y, \bar{z}) \leftrightarrow \alpha(x, y, \bar{z})),$$

then

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

For the third item, we need to justify the claim that if

$$(\forall x y \bar{z} \in M)(\alpha^M(x, y, \bar{z}) \leftrightarrow \alpha(x, y, \bar{z})),$$

then

$$(\forall y \bar{z} \in M) \begin{array}{c} (\forall x \in M)((x \in y) \rightarrow \alpha^M(x, y, \bar{z})) \\ \updownarrow \\ (\forall x)((x \in y) \rightarrow \alpha(x, y, \bar{z})). \end{array}$$

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. $\checkmark$
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. $\checkmark$
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

For the third item, we need to justify the claim that if

$$(\forall x y \bar{z} \in M)(\alpha^M(x, y, \bar{z}) \leftrightarrow \alpha(x, y, \bar{z})),$$

then

$$(\forall y \bar{z} \in M) \begin{array}{c} (\forall x \in M)((x \in y) \to \alpha^M(x, y, \bar{z})) \\ \updownarrow \\ (\forall x)((x \in y) \to \alpha(x, y, \bar{z})). \end{array}$$

$y \in M$, $x \in M$ and $x \in y$ is equivalent to $x \in y \in M$ since M is transitive.

Δ_0-Absoluteness

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

For the third item, we need to justify the claim that if

$$(\forall x y \bar{z} \in M)(\alpha^M(x, y, \bar{z}) \leftrightarrow \alpha(x, y, \bar{z})),$$

then

$$(\forall y \bar{z} \in M) \begin{array}{c} (\forall x \in M)((x \in y) \to \alpha^M(x, y, \bar{z})) \\ \updownarrow \\ (\forall x)((x \in y) \to \alpha(x, y, \bar{z})). \end{array}$$

$y \in M$, $x \in M$ and $x \in y$ is equivalent to $x \in y \in M$ since M is transitive.✓

Theorem. Δ_0-formulas are absolute for transitive classes.

Proof: Let M be a transitive class. It suffices to show that

1. Atomic formulas are absolute. ✓
2. If α and β are absolute, then $\neg\alpha$ and $\alpha \wedge \beta$ are absolute. ✓
3. If α is absolute, then $(\forall x \in y)\alpha$ is absolute.

For the third item, we need to justify the claim that if

$$(\forall x y \bar{z} \in M)(\alpha^M(x, y, \bar{z}) \leftrightarrow \alpha(x, y, \bar{z})),$$

then

$$(\forall y \bar{z} \in M) \begin{array}{c} (\forall x \in M)((x \in y) \rightarrow \alpha^M(x, y, \bar{z})) \\ \updownarrow \\ (\forall x)((x \in y) \rightarrow \alpha(x, y, \bar{z})). \end{array}$$

$y \in M$, $x \in M$ and $x \in y$ is equivalent to $x \in y \in M$ since M is transitive.✓

□

Facts about absoluteness

Facts about absoluteness

1. (Proposition 13.4 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N,

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition:

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute,

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST) In particular, the following are absolute for transitive class models of ZF:

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST) In particular, the following are absolute for transitive class models of ZF:
    1. ordinal arithmetic

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST) In particular, the following are absolute for transitive class models of ZF:
    1. ordinal arithmetic
    2. $\mathrm{rank}(x)$

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST) In particular, the following are absolute for transitive class models of ZF:
   1. ordinal arithmetic
   2. rank(x)
   3. tr.cl(x)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST) In particular, the following are absolute for transitive class models of ZF:
    1. ordinal arithmetic
    2. rank(x)
    3. tr.cl(x)
5. (Theorem 13.13, NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST) In particular, the following are absolute for transitive class models of ZF:
    1. ordinal arithmetic
    2. rank(x)
    3. tr.cl(x)
5. (Theorem 13.13, NST)

Facts about absoluteness

1. (Proposition 13.4 of NST) If $M \subseteq N$ are class models and F is a class function defined in both M and N by a formula that is absolute for M, N, then F has the same meaning in M and N (= computes the same value in M as in N on inputs from M).
2. (Theorem 13.7 of NST) Absoluteness is preserved by composition: If $\varphi(x_1, \ldots, x_n)$, $F(x_1, \ldots, x_n)$, and $G_i(\bar{y})$ are absolute, then so are $\varphi(G_1, \ldots, G_n)$ and $F(G_1, \ldots, G_n)$.
3. (Theorem 13.11 of NST) A function defined by recursion in a transitive class model M using absolute concepts will be absolute.
4. (Theorem 13.12 of NST) In particular, the following are absolute for transitive class models of ZF:
    1. ordinal arithmetic
    2. rank(x)
    3. tr.cl(x)
5. (Theorem 13.13, NST) If M is a transitive model of ZF, then $\mathcal{P}^M(x) = \mathcal{P}^V(x) \cap M$ and $V_\alpha^M = V_\alpha \cap M$.

Some properties that are not absolute for transitive models

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.
4. $|x| = \aleph_0$.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.
4. $|x| = \aleph_0$.
5. x is a cardinal.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.
4. $|x| = \aleph_0$.
5. x is a cardinal.
6. x is a regular cardinal.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.
4. $|x| = \aleph_0$.
5. x is a cardinal.
6. x is a regular cardinal.
7. x a strongly inaccessible cardinal.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.
4. $|x| = \aleph_0$.
5. x is a cardinal.
6. x is a regular cardinal.
7. x a strongly inaccessible cardinal.

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.
4. $|x| = \aleph_0$.
5. x is a cardinal.
6. x is a regular cardinal.
7. x a strongly inaccessible cardinal. (i.e., x is a regular cardinal, and $|y| < |x|$ implies $|\mathcal{P}(y)| < |x|$).

Some properties that are not absolute for transitive models

1. ($\neg$ AC) x is a nonempty partition that has no choice function.
2. x is amorphous.
3. x is Dedekind finite.
4. $|x| = \aleph_0$.
5. x is a cardinal.
6. x is a regular cardinal.
7. x a strongly inaccessible cardinal. (i.e., x is a regular cardinal, and $|y| < |x|$ implies $|\mathcal{P}(y)| < |x|$).
8. (GCH) For any infinite x and y, if $|x| \leq |y| \leq |\mathcal{P}(x)|$, then $|x| = |y|$ or $|y| = |\mathcal{P}(x)|$.