

Category Theory

Homework Assignment IX

For Alessandra, Daniel, Storey, Noah: Present the problems below on December 11.

PROBLEMS

1. Let $\pm\text{Grp}$ be the category of groups with \pm morphisms. By a $+$ -morphism I mean an ordinary group homomorphism $f(xy) = f(x)f(y)$, $f(x^{-1}) = f(x)^{-1}$, $f(e) = e$, while by a $-$ -morphism I mean an antihomomorphism $f(xy) = f(y)f(x)$, $f(x^{-1}) = f(x)^{-1}$, $f(e) = e$.

- (a) Does $\pm\text{Grp}$ have products? Equalizers?
- (b) Does the forgetful functor $U: \pm\text{Grp} \rightarrow \text{Set}$ have a left adjoint?

2. Let G be a group considered as a 1-object category, \mathcal{G} . Describe the possible monads on \mathcal{G} .

3. Exercise V.9.5 of Mac Lane.

4. It is a theorem of group theory that a Grp -morphism $\alpha: \mathbb{H} \rightarrow \mathbb{K}$ is an isomorphism if and only if the underlying Set -morphism $\alpha: H \rightarrow K$ is an isomorphism. (That is, a group homomorphism is an isomorphism if and only if it is bijective.) Show that if T is any monad on a category \mathcal{D} , then a morphism $\alpha: \mathbb{H} \rightarrow \mathbb{K}$ of T -algebras in \mathcal{D} is an isomorphism if and only if the underlying \mathcal{D} -morphism, $\alpha: H \rightarrow K$, is an isomorphism.