

Category Theory

Homework Assignment VIII

For everyone: Read Chapter VI of Mac Lane and Chapter 5 of Riehl.

For Nick, Alessandra, Gregory: Present the problems below on November 20.

PROBLEMS

1. Let $\pm\text{Grp}$ be the category of groups with \pm -morphisms. By a $+-$ -morphism, I mean an ordinary group homomorphism

$$f(xy) = f(x)f(y), f(x^{-1}) = f(x)^{-1}, f(e) = e,$$

while by a $--$ -morphism I mean an antihomomorphism

$$f(xy) = f(y)f(x), f(x^{-1}) = f(x)^{-1}, f(e) = e.$$

A \pm -morphism is either a $+-$ -morphism or a $--$ -morphism.

(a) Does $\pm\text{Grp}$ have products? Equalizers?

(b) Does the forgetful functor $U: \pm\text{Grp} \rightarrow \text{Set}$ have a left adjoint?

2. Exercise VI.2.1 (a), (b), (c) of Mac Lane.

3. Exercise 5.2.iv of Riehl.