Category Theory
Homework Assignment II

For everyone: Read Sections 2.1-2.5.

For Gregory, Tuscany, Khizar, Storey: Present the problems below on September 20.

PROBLEMS

1. Show that the forgetful functor $U\colon \mathbf{Grp} \to \mathbf{Set}$ is representable.

2. Suppose that a morphism $f\colon A \to B$ in $\mathcal{C}$ can be factored as $f = m \circ e$ where e is a split epi and m is monic. Show that this factorization is unique in the sense that if there is another such factorization $f = m' \circ e'$ then there is an isomorphism $d\colon X \to Y$ such that both triangles commute in

$$
\begin{array}{ccc}
A & \xrightarrow{\ e\ } & X \\
{\scriptstyle e'}\downarrow & {\scriptstyle d}\swarrow & \downarrow{\scriptstyle m} \\
Y & \xrightarrow[\ m'\ ]{} & B
\end{array}
$$

3. Let $\mathcal{C}$ be a category with finite products. The n-power functor from $\mathcal{C}$ to itself takes A to A^n. A *natural n-ary operation* on $\mathcal{C}$ is a natural transformation from the n-power functor to the identity functor.

   (a) Show that $+$ defines a natural binary operation on Ab.

   (b) Show that the only natural binary operations on Set are the projection operations: $\sigma_X(a, b) = a$ or $\tau_X(a, b) = b$.

4. Is Set equivalent to $\mathbf{Set}^{\mathrm{op}}$? Explain.