Truth tables! (Read Section 3.1 of the book.)

You should memorize the tables for $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

p	$\neg p$
0	1
1	0

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	0
1	1	1	1	1	1

You should be able to create tables for compound truth functions (or "propositions"), such as

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r) :$$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Please be able to explain/define the following.

(1) Tautology.

(2) Contradiction.

(3) Logically equivalent propositions.

(4) Logically independent propositions.

(5) Contrapositive (of $p \rightarrow q$).

(6) Converse (of $p \rightarrow q$).

(7) Inverse (of $p \rightarrow q$).

(8) Disjunctive normal form (and conjunctive normal form).

(9) Complete set of connectives.

Practice!

(1) Show that the following pairs of propositions are logically equivalent.

(a) $p \vee p$ and p.
(b) $\neg(\neg p))$ and p.
(c) $p \to q$ and $(\neg p) \vee q$.
(d) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$.
(e) (DeMorgan's Laws)
  (i) $\neg(p \wedge q)$ and $(\neg p) \vee (\neg q)$.
  (ii) $\neg(p \vee q)$ and $(\neg p) \wedge (\neg q)$.

(2) Consider the following two statements:

(a) "If we tip the waiter in advance, then we will get good service."

(b) "If we do not tip the waiter in advance, then we will not get good service."

Are these statements logically equivalent? Are they logically independent? Are there any possibilities other than logical equivalence or logical independence?

Let p be "we tip the waiter in advance" and let q be "we will get good service". The first proposition is $p \to q$, while the second is $(\neg p) \to (\neg q)$, which is the inverse statement. Thus, the two statements are logically independent. [If $p = 0$, $q = 1$, then the first is true and the second false, while if $p = 1$, $q = 0$, then the first is false and the second is true.]

(3) Is it a tautology to say "The Beatles were the best band of all time"?

This is not a tautology. In fact, it is not a compound proposition at all, so we would represent it as just p, which is not a tautology.

An example of a tautology built from this statement is $p \vee (\neg p)$, which in words is "The Beatles were the best band of all time, or they were not the best band of all time."

For the next three problems let p be "It is raining." and let q be "The ground is wet."

(4) Give an example of a sentence involving p and q that is a tautology.

(5) Give an example of a sentence involving p and q that is a contradiction.

(6) Give examples of two different sentences involving p and q that are logically equivalent.