Propositional logic

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- W = "The ground is wet".

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R	W	$R \wedge W$
0	0	0
0	1	0
1	0	0
1	1	1

R	W	$R \vee W$
0	0	0
0	1	1
1	0	1
1	1	1

R	$\neg R$
0	1
1	0

R	W	$R \rightarrow W$
0	0	1
0	1	1
1	0	0
1	1	1

R	W	$R \leftrightarrow W$
0	0	1
0	1	0
1	0	0
1	1	1

R	W	$R \oplus W = R \veebar W$
0	0	0
0	1	1
1	0	1
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Write the truth table for $P := R \vee (((A \wedge B) \to (\neg R)))$.

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Write the truth table for $P := R \vee (((A \wedge B) \rightarrow (\neg R)))$.

A	B	R	$A \wedge B$	$\neg R$	$((A \wedge B) \rightarrow (\neg R))$	P
0	0	0	0	1	1	1
0	0	1	0	0	1	1
0	1	0	0	1	1	1
0	1	1	0	0	1	1
1	0	0	0	1	1	1
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This proposition P is a **tautology**, because it assumes the value "true" under any truth assignment to the propositional variables.

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This proposition P is a **tautology**, because it assumes the value "true" under any truth assignment to the propositional variables. This means that P is true because of its logical structure alone, and not because of the truth values of its variables.

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Two propositions are **(logically) equivalent** if they assume the same truth value under any truth assignment to the propositional variables.
(Write $P \equiv Q$.)

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If X and Y are compound propositions, then $X = Y$ means that they are syntactically equal, while $X \equiv Y$ means that they are semantically equal.

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The truth table of a monomial has exactly one row whose value is $T = 1$:

A	B	C	D	$(\neg A) \wedge B \wedge C \wedge (\neg D)$
0	0	0	0	0
				$\vdots$
0	1	1	0	1
				$\vdots$
1	1	1	1	0

The monomial $(\neg A) \wedge B \wedge C \wedge (\neg D)$ assumes value 1 if and only if $A = 0, B = 1, C = 1, D = 0$.

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