Formal versus informal proofs!

Informal proof:

Theorem X. If A is nonempty, then it is possible to choose an element of A.
(Or, if $\exists x(x \in A)$, then there is a function $f : \{A\} \to A$.)

Proof. By the Axiom of Pairing, $\{A\}$ is a set. If $\exists x(x \in A)$, then by four more applications of the Axiom of Pairing $f := \{\{\{A\}, \{A, x\}\}\} = \{(A, x)\}$ is the desired function. $\square$

Proof using the formal proof management system, Coq.

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Theorem X:
forall (A: Type) (P: A-> Prop),
  (exists (a: A), P a)->
  (exists (f: unit-> A), P (f tt)).
Proof.
intros A P Pnonempty.
destruct Pnonempty as [awitness evidence].
exists (fun (x: unit)=> awitness).
exact evidence.
Qed.

X =
fun (A : Type) (P : A -> Prop)
  (Pnonempty : exists a : A, P a) =>
match Pnonempty with
| ex_intro _ awitness evidence =>
    ex_intro (fun f : unit -> A => P (f tt))
      (fun _ : unit => awitness) evidence
end
     : forall (A : Type) (P : A -> Prop),
       (exists a : A, P a) ->
       exists f : unit -> A, P (f tt)

Argument scopes are [type_scope function_scope _]
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