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Note that $(\exists x)\varphi(x)$ is true in $\mathbb{R}$ but false in $\mathbb{N}$. We learn if $(\exists x)(x < 0)$ is true in a structure by examining the table for “$<$”.

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We will describe a process to determine the truth of a sentence in a structure if the sentence is written in prenex form.

Interaction of $\forall$, $\exists$ with $\neg$, $\wedge$, $\vee$

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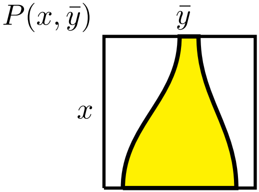

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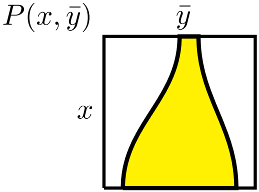

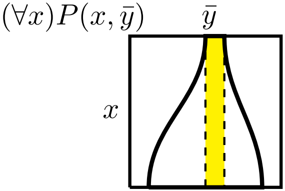

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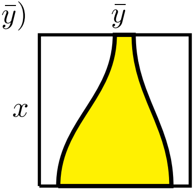

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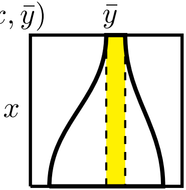

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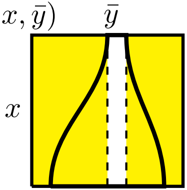

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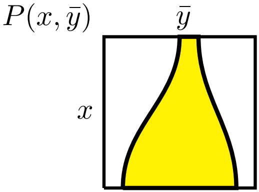

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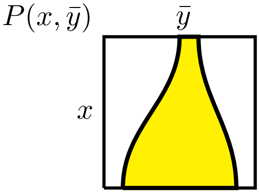

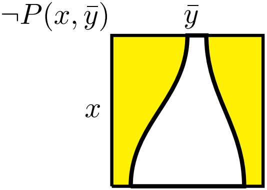

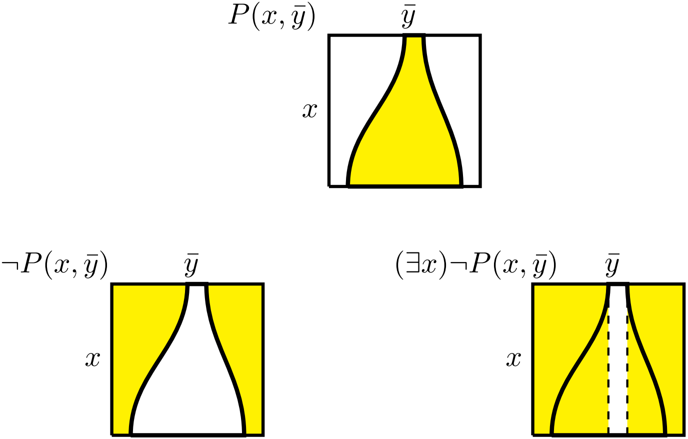
$P(x,\bar{y})$
$\bar{y}$
x
$\neg P(x,\bar{y})$
$\bar{y}$
x
$(\exists x)\neg P(x,\bar{y})$
$\bar{y}$
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