Practice with Inclusion/Exclusion, Stirling, and Bell numbers!

(1) Let $m = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 = 969969$ be the product of the first 8 distinct prime numbers. How many ways are there to factor m? Here, a factorization of m is a representation of m as a product of natural numbers greater than 1, as in $m = 30 \cdot 323 \cdot 1001$. (Assume that the order of the factors does not matter, so $m = 30 \cdot 323 \cdot 1001$ and $m = 1001 \cdot 323 \cdot 30$ are the same factorization.)

(2) In a class of 20 students, how many study groups can be formed which include at least one of the three students Archibald, Beryl, or Cornelia? Assume that a study group must involve at least 2 students.

(3) How many 6-digit numbers have the property that, for every k, the kth digit is different than the $(7-k)$th digit?

(4) A news organization reports that the percentage of voters who would be satisfied with candidates A, B, C for President is 65%, 57%, 58% respectively. Furthermore, 28% would accept A or B, 30% would accept A or C, 27% would accept B or C, and 12% would accept any of the three. Is this fake news?

(5) If $f : k \to k$ is a bijection, then i is called a fixed point of f if $f(i) = i$. What percentage of bijections $f : k \to k$ have no fixed points? (Count the number of bijections with no fixed points, then divide by the total number of of bijections.)

(6) Explain why $S(n, 2) = 2^{n-1} - 1$ if $n > 0$.

(7) Explain why $S(n, n-1) = \binom{n}{2}$.

(8) Assume that $|A| = 10$. How many equivalence relations on A have 5 equivalence classes?

(9) How many solutions are there to $x_1 + x_2 + x_3 + x_4 = 25$ if each x_i must be a natural number from the interval $[0, 10]$?