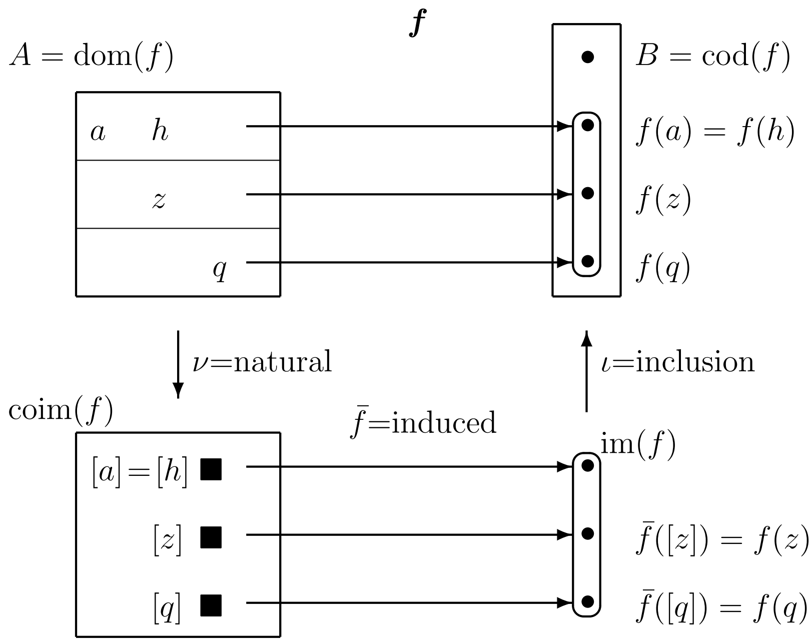
Terminology for functions.

Let A and B be sets and let $f\colon A \to B$ be a function from A to B. There are sets and functions related to A, B and f that have special names.



(1) The *image* of f is $\mathrm{im}(f) = f[A] = \{b \in B \;:\; \exists a \in A(f(a) = b)\}$. The image of a subset $U \subseteq A$ is $f[U] = \{b \in B \;:\; \exists u \in U(f(u) = b)\}$.
(2) The *preimage* or *inverse image* of a subset $V \subseteq B$ is $f^{-1}[V] = \{a \in A \;:\; f(a) \in V\}$.
(3) The preimage of a singleton $\{b\}$ is written $f^{-1}(b)$ and sometimes called the *fiber* of f *over* b. The fiber containing the element a is sometimes written $[a]$.
(4) The *coimage* of f is the set $\mathrm{coim}(f) = \{f^{-1}(b) \;:\; b \in \mathrm{im}(f)\}$ of all fibers.
(5) The *natural map* is $\nu\colon A \to \mathrm{coim}(f)\colon a \mapsto [a]$. (This says $\nu(a) = [a]$.)
(6) The *inclusion map* is $\iota\colon\ \mathrm{im}(f) \to B\colon b \mapsto b$. (This says $\iota(b) = b$.)
(7) The *induced map* is $\bar{f}\colon\ \mathrm{coim}(f) \to \mathrm{im}(f)\colon [a] \mapsto f(a)$. (This says $\bar{f}([a]) = f(a)$.)

Some facts:

(1) The natural map is *surjective*.
(2) The inclusion map is *injective*.
(3) The induced map is *bijective*.
(4) $f = \iota \circ \bar{f} \circ \nu$. (This is the *canonical factorization* of f.)