DISCRETE MATH
MIDTERM

Name:______________________________

You have 50 minutes for this exam. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

(1) Fill in the blank with all appropriate choices (if any exist).

(a) Naive set theory is <u>inconsistent</u>.

(i) free of any contradictions.

(ii) inconsistent.

(iii) called ZFC.

(b) The sentence

$$\forall x\, \forall y\, \exists p\, \forall z\, ((z \in p) \leftrightarrow (z = x \text{ or } z = y))$$

expresses the Axiom of <u>Pairing</u>.

(i) Extensionality

(ii) Power Set

(iii) Pairing

(c) By definition, a set is <u>infinite</u> if <u>it is not finite</u>.

(i) it has infinitely many elements.

(ii) it is not finite.

(iii) it has a bijection with ω.

(2) (a) What does it mean to say <u>f is a function from A to B</u>? (Give the definition.)

A function from A to B is a relation from A to B that satisfies the function rule.

(b) Define <u>inductive set</u>.
A set is inductive if it contains 0 and is closed under successor.

(c) Define <u>the set of natural numbers</u>.

The set of natural numbers is the set of elements common to all inductive sets.

(3) Give an example, if one exists, of each of the following. If no example exists, say why.

(a) A class that is not a proper class.

A class that is not a proper class is just a set. The simplest example is $\emptyset$.

(b) A partition of $\mathbb{N}$ with exactly two cells.

$\{\{0\}, \{n \in \mathbb{N} \mid n \neq 0\}\}$ is a partition of $\mathbb{N}$ into the singleton zero set and the set of nonzero natural numbers.

(c) A function between finite sets whose coimage is larger than its image.

No such function exists. If $f : A \to B$ is any function, then the induced map $\overline{f} : \mathrm{coim}(f) \to \mathrm{im}(f)$ is a bijection, so the coimage and image have the same cardinality.

(4) Let f be the function from $\{0, 1, 2\}$ to $\{a, b, c\}$ defined by $f(0) = f(1) = a$ and $f(2) = b$.

(a) Write $\mathrm{coim}(f)$ as a set.

$\{\{0, 1\}, \{2\}\}$.

(b) Write $\ker(f)$ as a set.

$\{(0, 0), (1, 1), (2, 2), (0, 1), (1, 0)\}$

(c) Write the induced map as a set.

$\{(\{0, 1\}, a), (\{2\}, b)\}$

(5) Write down the recursive definition of addition of natural numbers. Use the definition to show that $S(m) = m + 1$.

$$\begin{aligned} m + 0 &:= m && \text{(IC)} \\ m + S(n) &:= S(m + n) && \text{(RR)} \end{aligned}$$

For the second statement, for any m we have

$$\begin{aligned} m + 1 &= m + S(0) && \text{Defn of number 1} \\ &= S(m + 0) && \text{(RR)} \\ &= S(m) && \text{(IC)} \end{aligned}$$