Discrete Math
Quiz 8

Name:_____

You have 10 minutes to complete this quiz. You may not use any unauthorized sources and you may not communicate with others about the exam. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Write

($\diamond$) $$(((\exists x)P(x)) \to ((\forall x)Q(x)))$$

in prenex form.

$$\begin{aligned}
(((\exists x)P(x)) \to ((\forall x)Q(x))) &\equiv (((\exists x)P(x)) \to ((\forall y)Q(y))) && \text{Standardize variables} \\
&\equiv ((\neg(\exists x)P(x)) \vee ((\forall y)Q(y))) && \text{Prop. Logic} \\
&\equiv (((\forall x)\neg P(x)) \vee ((\forall y)Q(y))) && \neg\exists \equiv \forall\neg \\
&\equiv (\forall x)(\forall y)((\neg P(x)) \vee Q(y)) && \text{Move quantifiers to front}
\end{aligned}$$

Another valid answer is $(\forall x)(\forall y)(P(x) \to Q(y))$. (Use Propositional Logic to convert $(\neg P) \vee Q$ to $P \to Q$.)

2. Write an English sentence that has the logical structure of the displayed sentence ($\diamond$) from Problem 1.

If there is a leprechaun, then everyone has a chance of finding a pot of gold.

Here I am taking $P(x)$ to be "x is a leprechaun" and $Q(x)$ to be "x has a chance of finding a pot of gold".