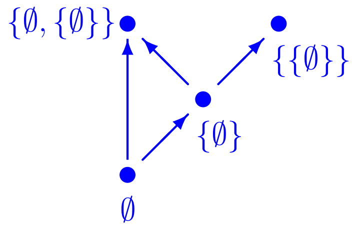
Solutions to HW 1.

1. Define $V_0 = \emptyset$, $V_1 = \mathcal{P}(V_0)$, $V_2 = \mathcal{P}(V_1)$, $V_3 = \mathcal{P}(V_2)$, and so on.
   (a) List the elements of V_0, V_1, V_2 and V_3.
      (i) $V_0 = \emptyset$,
      (ii) $V_1 = \mathcal{P}(\emptyset) = \{\emptyset\}$,
      (iii) $V_2 = \mathcal{P}(V_1) = \{\emptyset, \{\emptyset\}\}$,
      (iv) $V_3 = \mathcal{P}(V_2) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$,
   (b) Draw a directed graph whose "dots" are the sets in V_3 and where $x \to y$ means $x \in y$. (Hint: your graph should have four "dots" and four edges.)



2. Find sets A and B satisfying the given conditions.
   (a) $A \in B$ and $A \not\subseteq B$.
   There are many answers, such as $A = \{\emptyset\}$ and $B = \{\{\emptyset\}\}$.
   (b) $A \in B$ and $A \subseteq B$.
   You could take $A = \emptyset$ and $B = \{\emptyset\}$.
   (c) $A \notin B$ and $A \subseteq B$.
   You could take $A = B = \emptyset$.

3. Show that $\bigcup \mathcal{P}(x) = x$.

According to the Axiom of Extensionality, to show that the sets $\bigcup \mathcal{P}(x)$ and x are equal, we must show that they have the same elements.

Claim 1: We show that any element $y \in \bigcup \mathcal{P}(x)$ is an element of x.

If $y \in \bigcup \mathcal{P}(x)$, then y is an element of an element of $\mathcal{P}(x)$ according to the definition of union. This means that there is some set z such that $y \in z \in \mathcal{P}(x)$. Since $z \in \mathcal{P}(x)$, we know that $z \subseteq x$ according to the definition of $\mathcal{P}(x)$. But now we know that $y \in z$ and $z \subseteq x$, so we derive $y \in x$ from the definition of $\subseteq$.

Claim 2: We show that any element $y \in x$ is an element of $\bigcup \mathcal{P}(x)$.

Choose any $y \in x$. Let $z = \{y\}$ (which is a set according to the Pairing Axiom). Observe $z \subseteq x$ by the definition of subset. Now we have $y \in z$ and $z \subseteq x$, hence $y \in z \in \mathcal{P}(x)$. This yields $y \in \bigcup \mathcal{P}(x)$.