

Solutions to HW 6.

1. Write the following propositions in disjunctive normal form, assuming that each proposition is a function of p , q and r .

Note: In DNF, every variable should appear in every monomial, and the number of monomials should equal the number of 1's in the truth table of the proposition.

(i) $p \rightarrow r$

$$((\neg p) \wedge (\neg q) \wedge (\neg r)) \vee ((\neg p) \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge q \wedge r)$$

(ii) $((p \rightarrow q) \rightarrow ((\neg p) \leftrightarrow r))$.

$$((\neg p) \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge q \wedge (\neg r))$$

(iii) q

$$((\neg p) \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge q \wedge (\neg r)) \vee (p \wedge q \wedge r)$$

2. Write the following axioms of set theory as formal sentences.

(i) Extensionality.

$$(\forall A)(\forall B)((A = B) \leftrightarrow (\forall z)((z \in A) \leftrightarrow (z \in B)))$$

(ii) Pairing.

$$(\forall A)(\forall B)(\exists P)(\forall z)((z \in P) \leftrightarrow ((z = A) \vee (z = B)))$$

(iii) Power set.

$$(\forall A)(\exists P)(\forall z)((z \in P) \leftrightarrow \underline{\underline{z \subseteq A}})$$

or

$$(\forall A)(\exists P)(\forall z)((z \in P) \leftrightarrow \underline{\underline{(\forall w)((w \in z) \rightarrow (w \in A))}})$$

3. In 1959, Pete Seeger took lines from the Book of Ecclesiastes to write a song, which was made famous by the Byrds in 1965. One line is:

To every thing there is a season, and a time to every purpose under heaven.

Write this as a formal sentence using predicates $S(s, t) =$ “ s is the season for thing t ” and $P(T, p) =$ “ T is the time for purpose p ”.

$$((\forall t)(\exists s)S(s, t)) \wedge ((\forall p)(\exists T)P(T, p))$$