Nick Jamesson
Bob Kuo
Dan Lyness

6. Show that for any torsion free abelian group A, the first Ulm subgroup of A is divisible.

Proof. Let A be a torsion free abelian group, where A' is the first Ulm subgroup of A. Choose any $a \in A'$ and choose any $n \in \mathbb{N}$. Since $a \in A'$ there exists some $x \in A$ such that $a = nx$. We must show that $x \in A'$, which is equivalent to showing that x is divisible by every positive integer. Pick some arbitrary positive integer m.

Since $a \in A'$, there exists some $y \in A$ such that $a = nmy$. Then:

$$
\begin{aligned}
& nx = a = nmy & \\
\implies \quad & n(x - my) = 0 & \\
\implies \quad & x - my = 0 & \text{(since } A \text{ is torsion free)} \\
\implies \quad & x = my &
\end{aligned}
$$

Hence x is divisible by m, so since m was arbitrary, x is divisible by every positive integer, which implies that $x \in A'$. Thus for every $a \in A'$ and $n \in \mathbb{N}$ we have found some $x \in A'$ such that $a = nx$. This shows that A' is divisible. $\square$