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1. Show that a direct product of injective R-modules is injective.

Proof. Let $\{Q_i\}$ be injective R-modules. Let $Q = \prod_{i \in I} Q_i$. Note that Q is also an R-module. Let M be an arbitrary R-module with N an R-submodule of M such that there exists $\varphi : N \to Q$. Let $\iota : N \to M$ be an inclusion map so that ι is injective. Now define $\pi_i : Q \to Q_i$ to be the projection R-module homomorphism. Note that $\pi_i \circ \varphi : N \to Q_i$ is an R-module homomorphism. Since Q_i is injective there exists $\psi_i : M \to Q_i$ such that $(\pi_i \circ \varphi) = \psi_i \circ \iota$. Let $\psi : M \to Q$ be defined by $\psi(m) = (\psi_i(m))_{i \in I}$. This is also an R-module homomorphism. Let $n \in N$ and observe

$$\begin{aligned}(\psi \circ \iota)(n) &= \psi(\iota(n)) \\ &= (\psi_i(\iota(n)))_{i \in I} \\ &= \varphi(n).\end{aligned}$$

So $\varphi = \psi \circ \iota$ and we have shown Q is injective. $\square$