Modules, II

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Concept	univ. prop.	**CRng**	k-**Alg**	R-modules
Product	—	$R \times S$	$R \times S$	$M \times N$
Coproduct	—	$R \otimes_{\mathbb{Z}} S$	$R \otimes_k S$	$M \oplus N$
Free object/$\{x\}$	—	$\mathbb{Z}[x]$	$k[x]$	${}_R R$
Free object/X	—	$\mathbb{Z}[X]$	$k[X]$	$\bigoplus_X {}_R R$

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Recall picture

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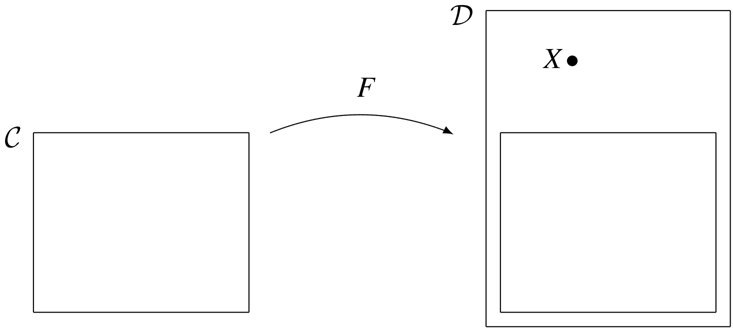

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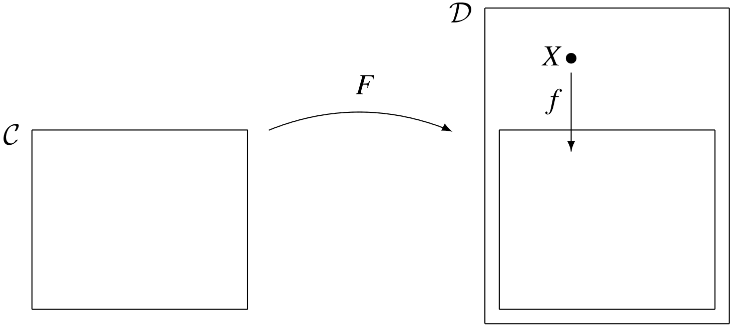

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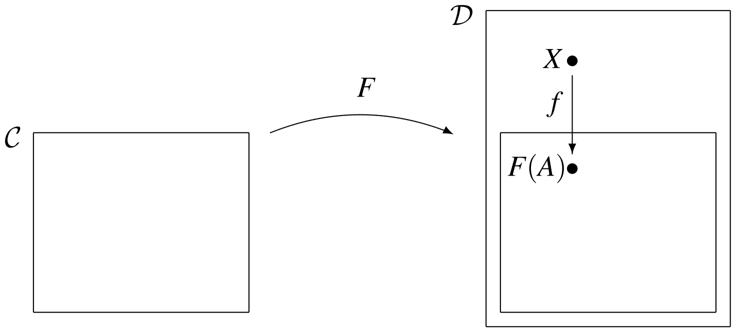

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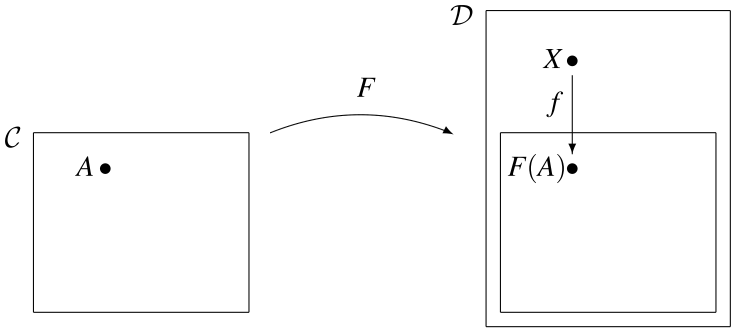

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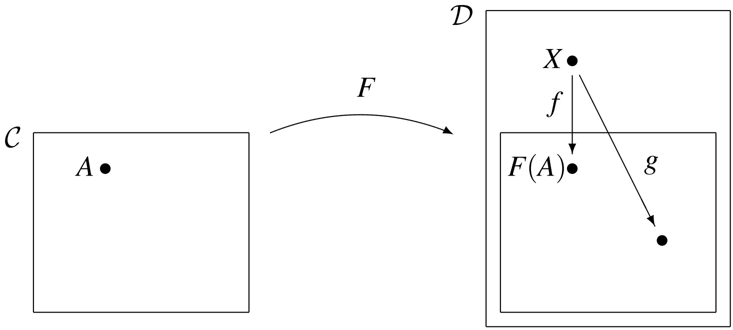

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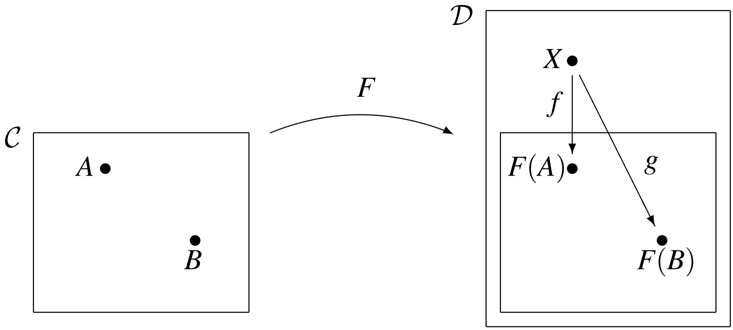

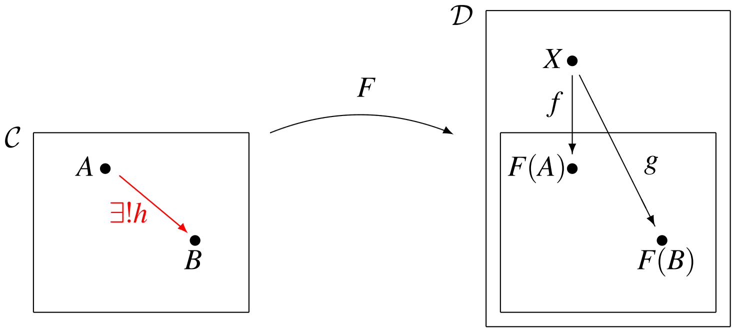
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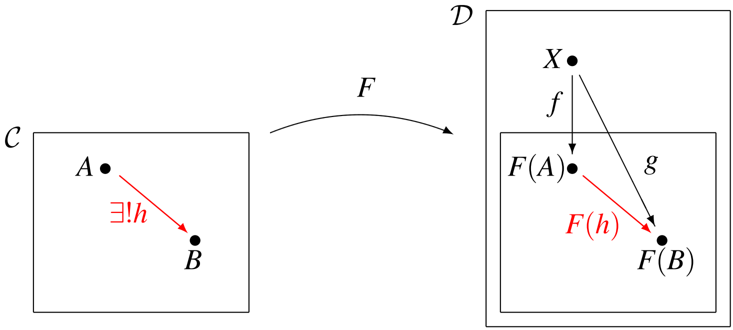

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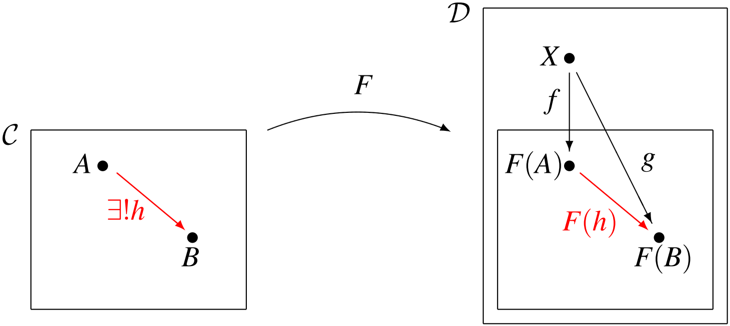
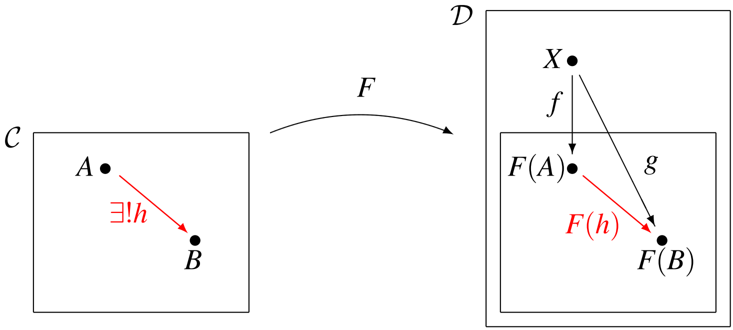
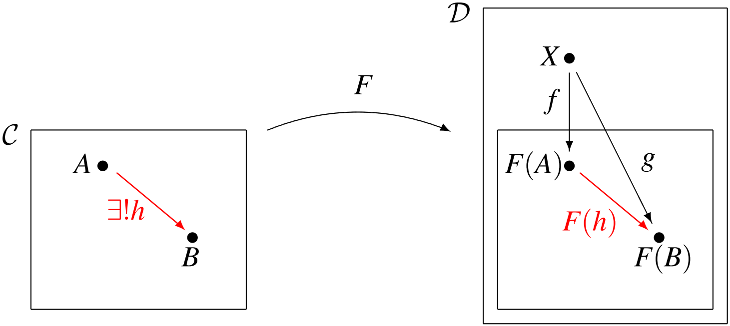


Let F = forgetful functor

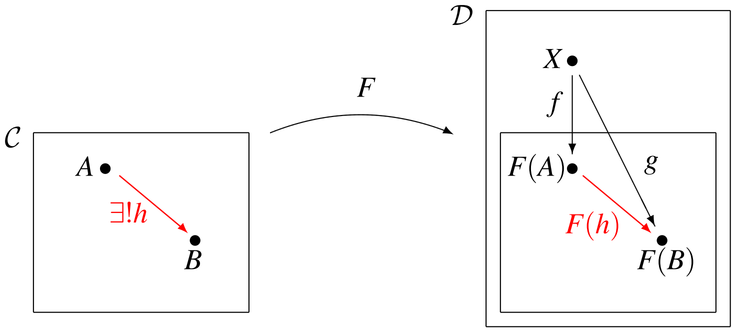
Recall picture



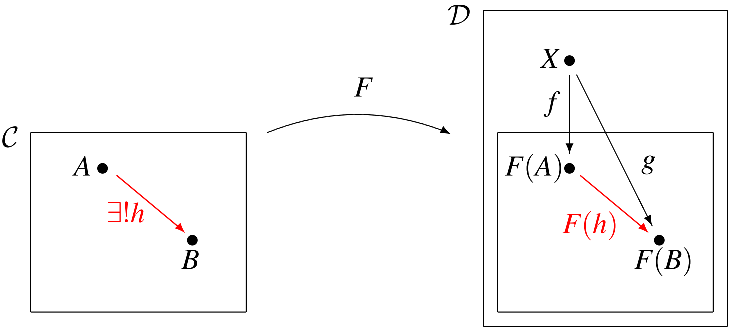
Let F = forgetful functor/underlying set functor



Let F = forgetful functor/underlying set functor to $\mathcal{D}$ = **Set**.



Let F = forgetful functor/underlying set functor to $\mathcal{D}$ = **Set**. Let $X = \{*\}$ be a 1-pt set.



Let F = forgetful functor/underlying set functor to $\mathcal{D}$ = **Set**. Let $X = \{*\}$ be a 1-pt set. If we have a universal morphism $f : X \to F(A)$, then A satisfies:

$f : X \to F(A)$ *selects a special point of the underlying set of* A,

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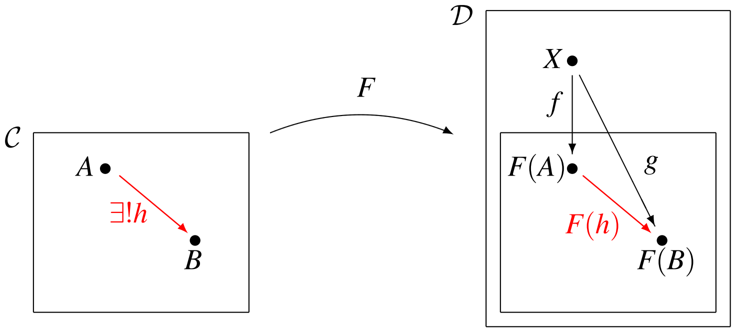

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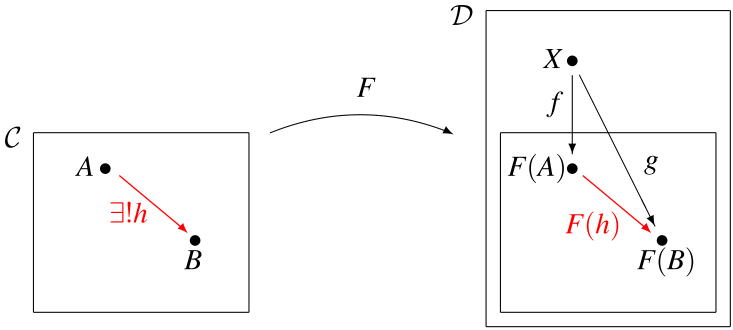

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> *$f : X \to F(A)$ selects a special point of the underlying set of A, such that any morphism $g : X \to F(B)$ selecting a point of the underlying set of some B*

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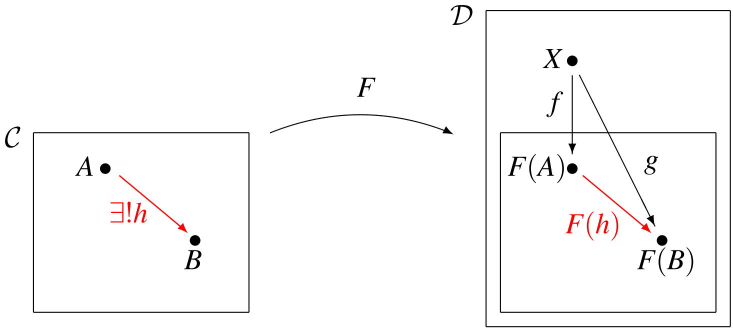

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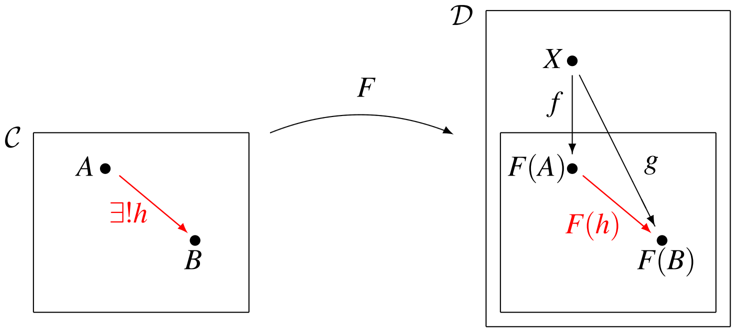
A is the "free $\mathcal{C}$-object over $X = \{*\}$".



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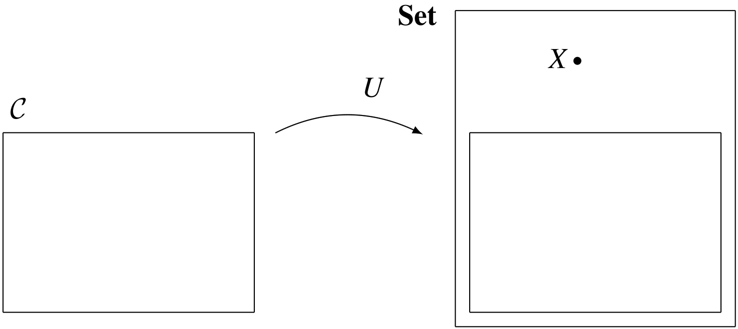
A is the "free $\mathcal{C}$-object over $X = \{*\}$". Or, A "represents the underlying set functor, $F(-) \cong \mathrm{Hom}_{\mathcal{C}}(A, -)$".

Free objects



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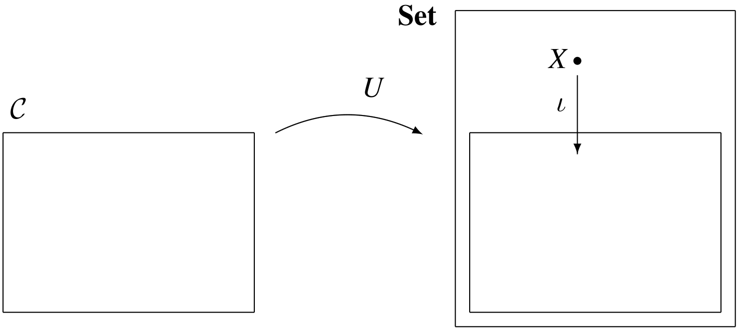

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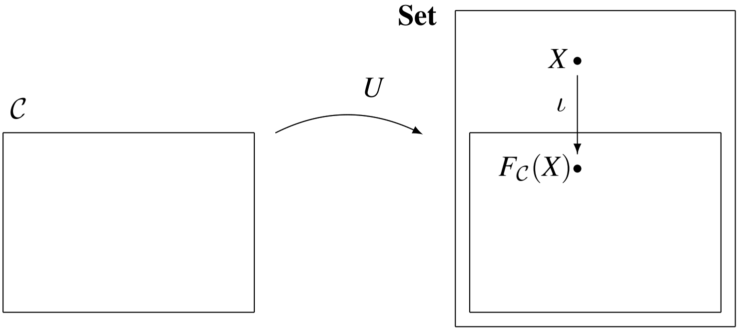

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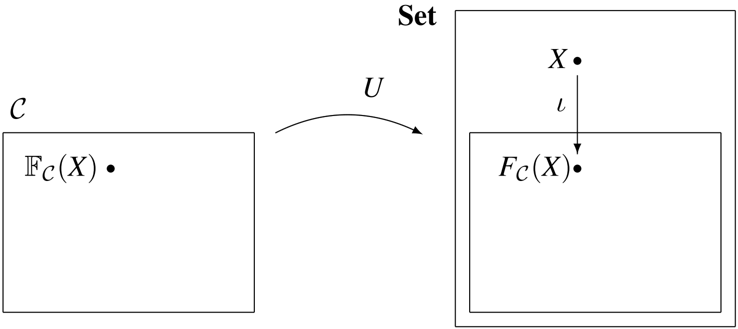

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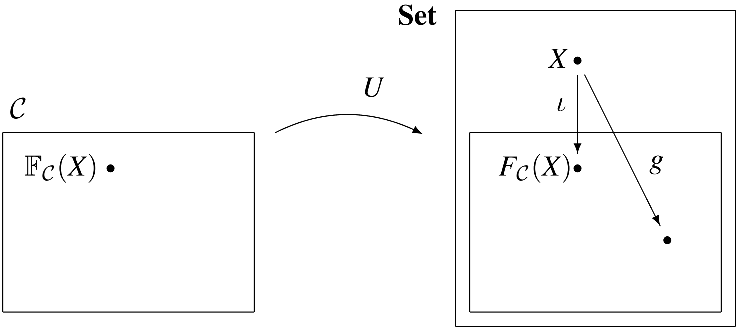

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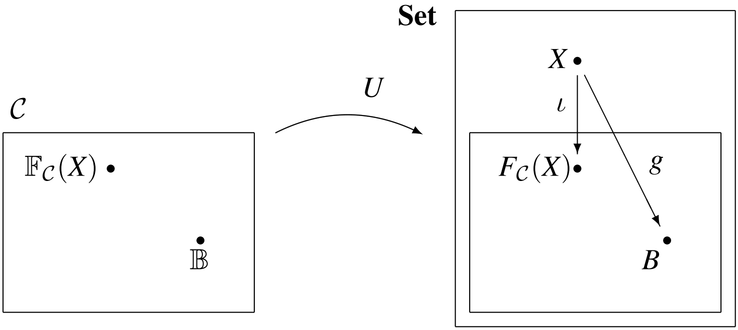
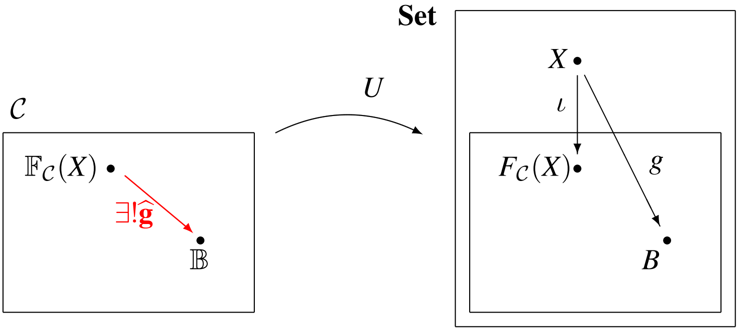


Free objects



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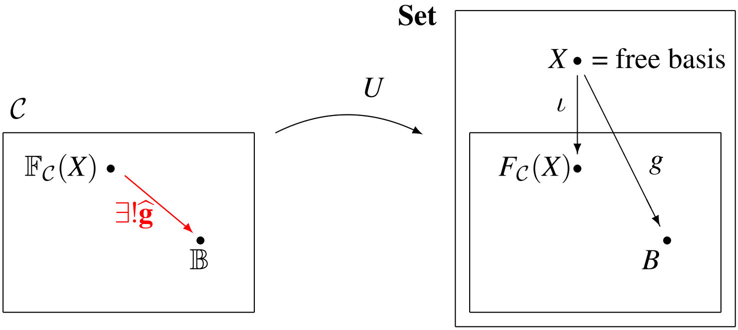

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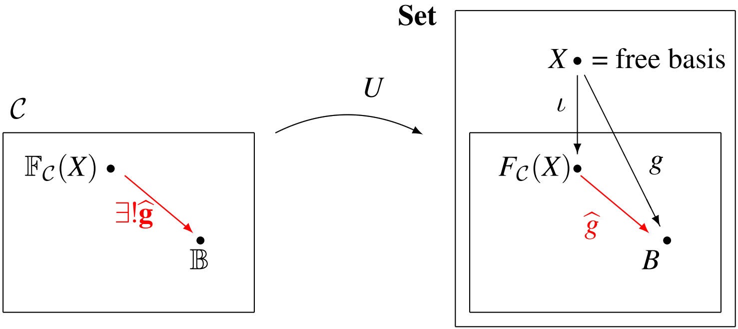

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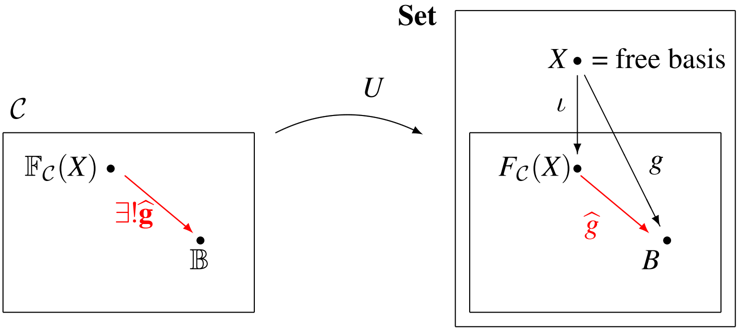
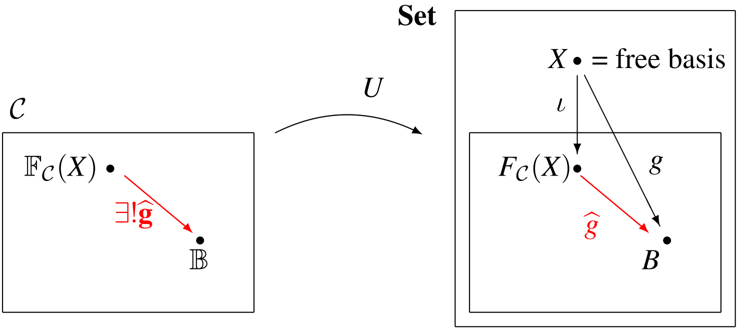


1. Free set over X = ?

Free objects



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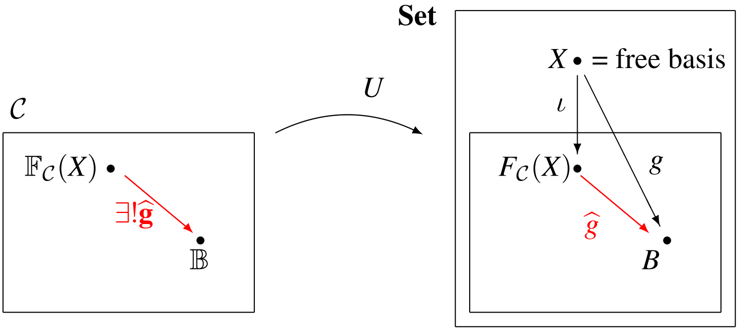
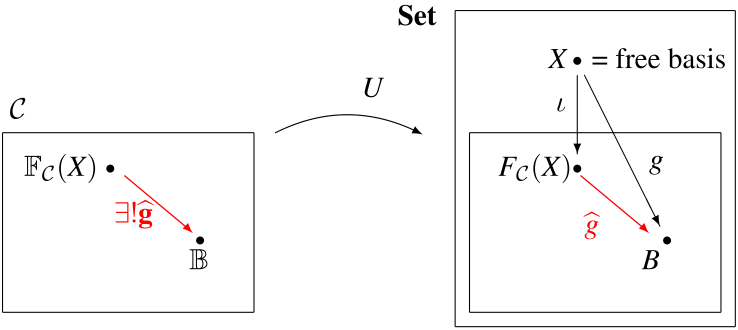


1. Free set over X = ?
2. Free topological space over X = ?

Free objects



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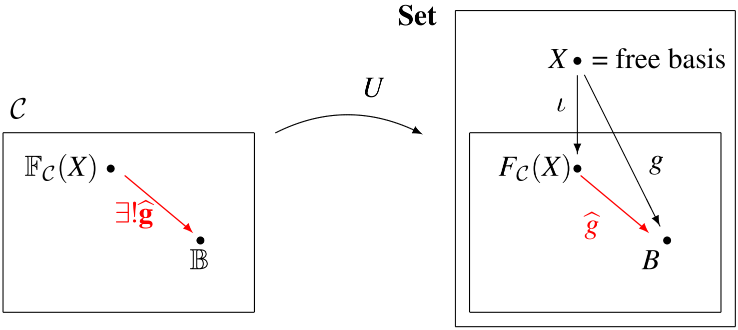

1. Free set over X = ?
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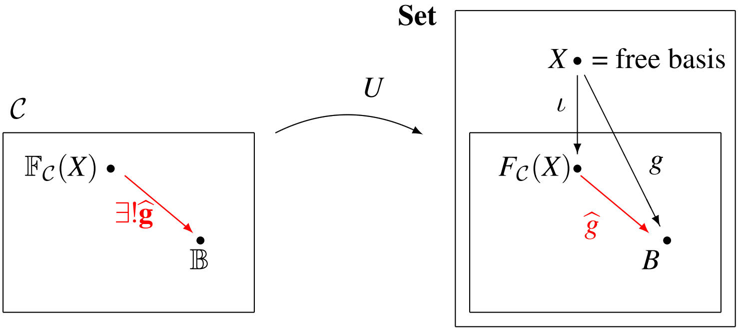
3. Free vector space over X = ?

Free objects



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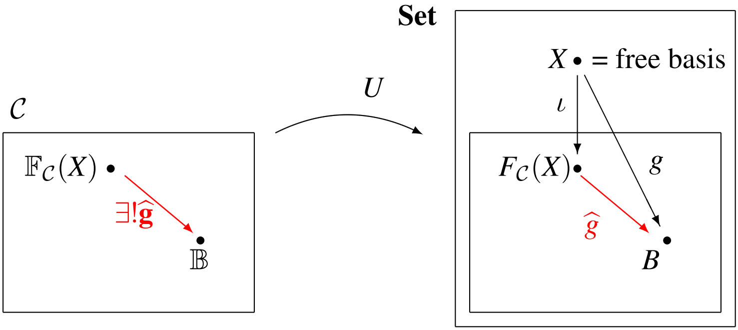
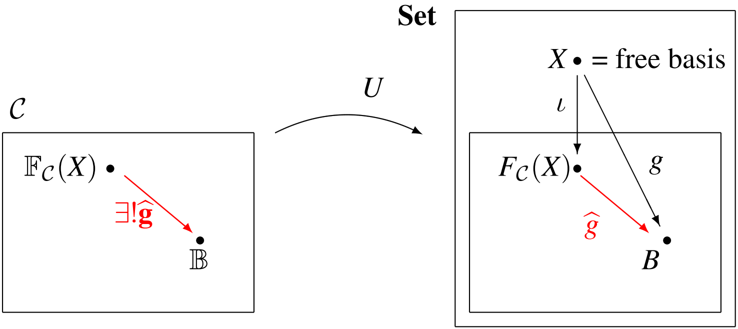
Free objects



1. Free set over $X = ?$
2. Free topological space over $X = ?$
3. Free vector space over $X = ?$
4. Free field over $X = ?$

Free objects



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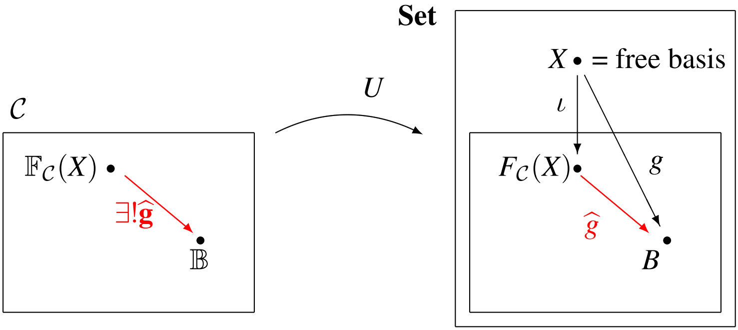
Free objects



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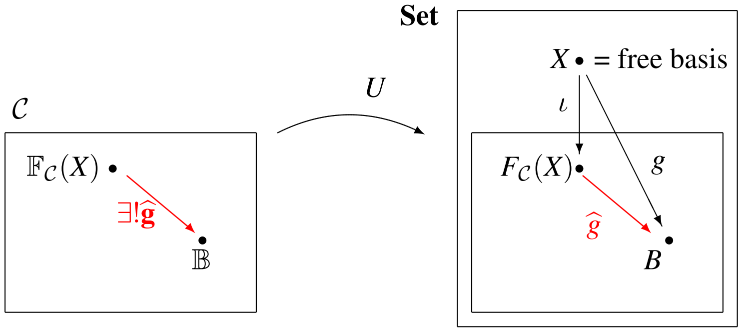
3. Free vector space over X = ?
4. Free field over X = ?
5. k-algebra over X = ?



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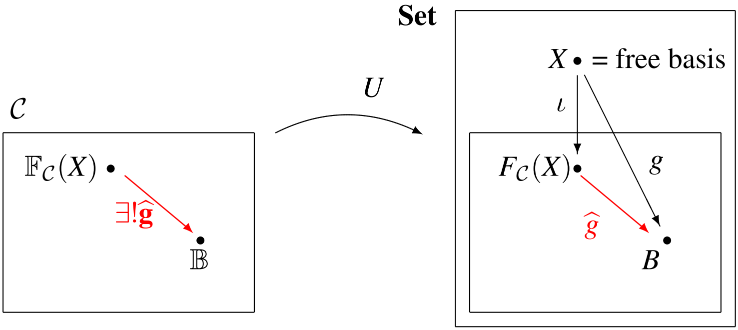

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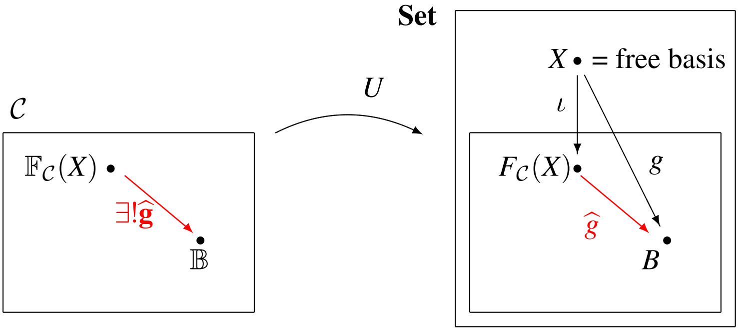
2. Free topological space over X = ?
3. Free vector space over X = ?
4. Free field over X = ?
5. k-algebra over X = ?
6. R-module over X = ?



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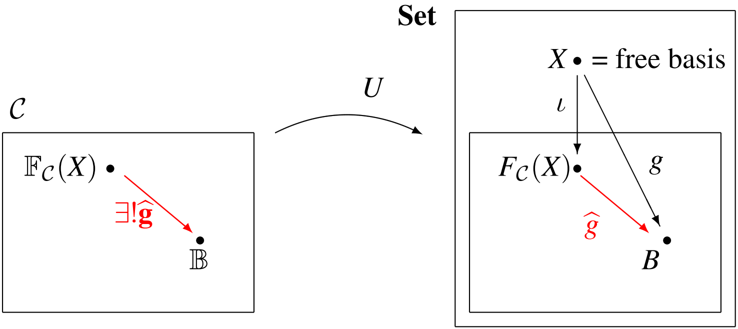
Free objects



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3. Free vector space over X = ?
4. Free field over X = ?
5. k-algebra over X = ?
6. R-module over X = ?