Warmup!

Let $A = \mathbb{Z}$, let $S = \{2\}^* = \{1, 2, 2^2, \ldots\}$, and consider $A \to S^{-1}A : a \mapsto a/1$.

(1) If $I = (6) \lhd \mathbb{Z}$, then what is $I^{ec} = (S^{-1}(6))|_{\mathbb{Z}} = \mathrm{sat}(I)$?

(2) Describe the ideal lattice of $S^{-1}A$.

(3) Interpret the answer to (2) as a statement about $\mathrm{Spec}(A)$ versus $\mathrm{Spec}(S^{-1}A)$.