**Set Theory**
**Quiz 9**

**Name:**____________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Define the Hartogs number, $h(X)$, of a set $X$.

$h(X)$ is the least ordinal not embeddable in $X$.

2. Give an example of a finite ordinal that is not a Hartogs number, and then give an example of an infinite ordinal that is not a Hartogs number.

- If $n \in \omega$, then $h(n) = n + 1$, so any finite successor ordinal is a Hartogs number. Consequently, the only finite ordinal that is not a Hartogs number is 0.
- Any Hartogs number is an initial ordinal, so $\omega+1$ cannot be a Hartogs number.

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Remark: $h(X) = \omega$ exactly when $X$ is an infinite *Dedekind finite* set. Any amorphous set is an infinite Dedekind finite. (Recall that $A$ is amorphous if $A$ is infinite but it cannot be partitioned into two infinite subsets.)

It is consistent with ZF that infinite Dedekind finite sets exist, and also consistent with ZF that infinite Dedekind finite sets do not exist. In ZFC, infinite Dedekind finite sets do not exist. Thus, $\omega$ cannot be a Hartogs number in ZFC, but it can be a Hartogs number in ZF.